Persuasion of Loss-Averse Receivers Through Early Offers*

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Abstract

We study a simple bargaining model in which the sender can make an early offer to the receiver. Initially, the sender has private information about the value of the receiver's outside option. The receiver learns this value before she chooses between the sender's early offer and her outside option. Nevertheless, if the receiver is expectations-based loss averse, the sender can persuade her to accept an offer that is inferior to her outside option. This result is due to the interaction of two effects: the *attachment effect* that makes it costly for the receiver to reject an offer that she planned to accept, and the *uncertainty effect* which renders the acceptance of the sender's offer as the preferred plan since it creates peace of mind at an early stage. If the receiver faces uncertainty in multiple dimensions, the main result holds for all degrees of loss aversion. Thus, expectations-based loss-averse preferences imply that there is scope for persuasion through signaling even if the receiver has all payoff-relevant information at the decision stage.

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1 Introduction

In many settings, a party with superior information tries to influence the choices of a less well-informed decision maker. Economists have extensively studied the scope for *persuasion* of rational decision makers through a variety of mechanisms: signaling (Spence, 1973), cheap talk (Crawford and Sobel, 1982), and Bayesian persuasion (Kamenica and Gentzkow, 2011). A common feature of these mechanisms is that, at the point in time when the decision maker chooses an option, she has incomplete information about the state of world (such as the opponent's type or the realization of a payoff-relevant state variable). She only can infer details about the state of the world based on the actions of or the information provided by the informed party. In an environment with complete information at the decision stage, however, there is no scope for persuasion of rational decision makers with standard preferences.

This changes when the decision maker (she) is rational, but exhibits reference-dependent preferences. The actions of the informed party (he) may endogenously change the decision maker's reference point, which in turn affects her preferences over options at the decision stage. Hence, the informed party may be able to influence the choices of the decision maker with reference-dependent preferences, even if she has complete information about the state of the world at the point in time when she makes a final decision. This type of persuasion does not require any congruence of preferences (as in cheap talk) or commitment on the side of the sender (as in Bayesian persuasion).

In this paper, we study persuasion through signaling to receivers with reference-dependent preferences. To this end, we consider a simple dynamic model of bargaining between a sender and a receiver. The receiver initially does not know the value of her outside option. She learns this value before she makes a choice. The sender knows the receiver's outside option value right away. Before the receiver learns about her outside option, the sender can make a binding "early offer" to the receiver, i.e., an offer that is valid in the last stage of the game when the receiver has complete information. The receiver then chooses between this early offer and her outside option. This interaction defines a signaling game since the sender's early offer may inform the receiver about her outside option value. To capture the receiver's reference-dependent preferences, we assume that she is expectations-based loss averse (Kőszegi and Rabin, 2006, 2007). After observing the receiver's early offer, she updates her beliefs about her outside option and makes a plan under what circumstances she accepts which option. Her beliefs and plan jointly determine her reference point. This reference point defines her preferences at the decision stage.

Under standard preferences, the sender would have to make an offer that is at least as good for the receiver as the outside option, otherwise the receiver would reject it. We show that, if the receiver is loss-averse, then an equilibrium may exist in which the sender persuades the receiver through signaling to accept an early offer that has a lower total value than her outside option (henceforth, an "inferior option"). The sender therefore can benefit from making early offers even when the receiver has all payoff-relevant information at the decision stage. Our framework captures different strategic situations.

Market Example. The sender is a telecommunication service provider and the receiver a consumer of mobile phone services. The receiver's contract with the sender is about to expire and, in anticipation of this event, the sender proposes a renewal contract. The sender has a competitive advantage relative to other providers: He knows the consumer's user profile and therefore can offer services at a lower price (or offer better services). He also knows the configuration of his rivals' contracts for new clients. In contrast, when observing the sender's renewal contract offer, the receiver does not yet know her utility from alternative contracts in the market. However, she learns this value (e.g., from a price comparison website) at a later stage before choosing between the sender's offer and the best alternative contract in the market.

Negotiation Example. The sender is the HR representative of a large firm and the receiver a young professional who recently graduated from university. Suppose the sender strictly prefers the young professional to other potential hires and the receiver strictly prefers working for the sender's firm to working at other companies in the industry. The sender also knows what contracts these other companies would offer to the receiver (in terms of compensation and working conditions). In contrast, the receiver does not yet know her "BATNA" – the best alternative to a negotiated agreement – when observing the receiver's offer.[1] However, the receiver will have more job interviews. Before making a final decision, she learns her BATNA and decides about whether to accept the sender's offer or the best alternative job option.

In order to persuade the receiver to accept an inferior option, the sender needs to differentiate his offer from the outside option, i.e., it needs to have a feature that the outside option does not have. We allow the total value of the early offer to consist of a regular value and a transfer. A loss-averse receiver treats the regular value and the transfer dimension separately.[2] The regular value occurs in the same dimension as the outside option value, so that these are directly comparable for the receiver. The transfer occurs in a dimension in which the outside option

[1]The BATNA is the value that the receiver would get if negotiations fail and no agreement is reached. This concept has been developed by Roger Fisher and William Ury and it is explained in detail in Fisher et al. (2011). The difference between the BATNA and a reservation value is that the latter is defined as the lowest value the receiver would be willing to accept.

[2]This separation is a feature of the expectations-based reference point model of Kőszegi and Rabin (2006, 2007) and is closely linked to mental accounting and the endowment effect. It describes individuals' tendency to assess gains and losses separately across different dimensions (Kahneman et al., 1990, 1991, Thaler, 1985, 1999).

only offers a zero outcome. Therefore, if the receiver is loss-averse, the sender can differentiate his offer from the outside option through the transfer without changing its total value.

As an illustration, suppose the early offer has a regular value below the outside option value and a positive transfer. In this case, if the receiver's reference point is defined by the plan "accept the early offer", then choosing the outside option creates a gain in the regular value dimension and a loss in the transfer dimension. Loss aversion (the tendency that losses loom larger than gains of similar size) then reduces the payoff from accepting the outside option. This enables the sender to redistribute surplus from the receiver to himself.

Whether such redistribution takes place in equilibrium depends on how the sender makes early offers to the receiver. Note that the receiver could just plan to reject the sender's offer and accept the outside option with certainty. Two different effects – induced by expectations-based loss-averse preferences – interact to enable persuasion through signaling: the *attachment effect* and the *uncertainty effect*. The *attachment effect* implies that it is costly for the receiver to choose the outside option, provided that accepting the early offer determines the reference point. As described above, this effect is caused by gain-loss sensations in the regular value and transfer dimension. The *uncertainty effect* makes the acceptance of the early offer relatively more attractive than the acceptance of the outside option. Planing the acceptance of the sender's offer creates peace of mind at an early stage, while planing the acceptance of the outside option exposes the receiver to uncertainty, which in turn lowers her expected payoff. Both the attachment and the uncertainty effect must be strong enough so that it is optimal for the receiver to plan the acceptance of and eventually accept an offer that with certainty is less valuable for her than the outside option.

We examine the structure of equilibria in which, for any realization of the outside option value, the sender persuades the receiver to accept an offer that is inferior to her outside option. When the receiver faces uncertainty only in one outcome dimension, such an equilibrium exists if the loss aversion parameters – the weight of gain-loss sensations η and the degree of loss aversion λ – are sufficiently large. We show that such an equilibrium must be a semi-separating signaling equilibrium. To generate an uncertainty effect, the sender's early offer informs the receiver about the range of possible outside option values. Thus, it has an interval structure, reminiscent of an equilibrium with information transmission in a cheap talk game (Crawford and Sobel, 1982). We show that any sender-preferred equilibrium exhibits this feature if there is uncertainty only in one outcome dimension.

In our baseline model, persuasion through signaling is possible if the loss aversion parameters are sufficiently large. Specifically, we need that $\eta(\lambda - 1) > 3$. As we discuss in the next section, these are empirically relevant degrees of loss aversion as there is substantial evidence for the uncertainty effect. Nevertheless, in most theoretical applications, the assumed levels of loss aversion are typically smaller. In some applications, large levels of loss aversion are ruled out explicitly, e.g., in Herweg and Mierendorff (2013).

We show that our main result can hold for all loss aversion parameter values η, λ that satisfy $\eta(\lambda - 1) > 0$ if the receiver faces uncertainty about the outside option in multiple outcome dimensions. For example, she may be uncertain about details of the product specification of the outside option, such as design, customer service, delivery times, or warranties. In contrast, she immediately observes all these details for the sender's early offer. Uncertainty in multiple dimensions implies that the plan "accept the outside option" exposes the receiver to gain-loss sensations even if the sender's offer would perfectly signal the total value of the outside option to the receiver at an early stage. It turns out that this feature of the environment allows the sender to persuade the receiver to accept an inferior offer, regardless of the degree of loss aversion. Thus, expectations-based loss-averse preferences imply that there is scope for persuasion through signaling even if the receiver has all payoff-relevant information at the decision stage.

The rest of the paper is organized as follows. In Section 2, we explain how our paper contributes to the related literature. In Section 3, we introduce the formal model and define the equilibrium concept. In Section 4, we derive our main result for the baseline model and characterize the sender-preferred equilibrium. In Section 5, we consider uncertainty in multiple dimensions and show that, in such a setting, our main result obtains for any positive degree of loss aversion. In Section 6, we examine a number of extensions and robustness checks of our baseline model. Section 7 concludes. All proofs are relegated to the appendix.

2 Related Literature

Strategic interaction of agents with expectations-based loss-averse preferences. Our paper mainly contributes to the literature that analyzes the implications of expectations-based loss aversion for strategic interaction. The most closely related papers in this literature study a monopolist's optimal pricing and marketing strategy when consumers are expectations-based loss averse. Herweg and Mierendorff (2013) consider a setting in which consumers face demand uncertainty. Loss-averse consumers may strictly prefer a flat rate tariff to a measured tariff even if it is not the option that minimizes their expected expenses. This makes it optimal for the monopolist to offer flat rate contracts. Heidhues and Kőszegi (2014) and Rosato (2016) show that a monopolist can exploit the consumers' loss aversion by creating attachment to its product through commitment to sophisticated price strategies: Heidhues and Kőszegi (2014) consider mixtures of sales and regular prices, Rosato (2016) quantity restrictions on products that are on sale. Karle and Schumacher (2017) demonstrate that a monopolist can also use the partial revelation of match value information to create consumer attachment. Further, Hancart (2023) shows that the monopolist may randomize over prices in equilibrium even if it cannot commit to its strategy, as in Heidhues and Kőszegi (2014).[3] Importantly, the results in these papers only rely on the attachment effect: A loss-averse consumer who expects to obtain a product will find it more difficult to decide against its purchase than a consumer who expects not to own it. The main contribution of the present paper in this literature is to consider the interaction of the attachment and the uncertainty effect.

Uncertainty Effect. Several experimental studies find versions of the uncertainty effect. Gneezy et al. (2006) first demonstrated that some individuals value a lottery less than its worst outcome. They apply a between-subject design and obtained the same result for different types of goods, elicitation methods, and implementation. Sonsino (2008) finds in auctions for single gifts and binary lotteries on these gifts that 27 percent of subjects sometimes submit higher bids for the single gift than for the lottery even though the lottery's worst outcome is the gift. In a post-experimental survey, many participants indicate "aversion to lotteries" as their explanation for such behavior. Simonsohn (2009) conducts several within-subject variations of the experiment by Gneezy et al. (2006) and finds that 62 percent of subjects exhibit the uncertainty effect. Newman and Mochon (2012) demonstrate that these results also hold in settings that largely avoid disappointments, i.e., there are different potential outcomes, but they are all valued roughly the same. Yang et al. (2013) show that a pronounced uncertainty effect occurs if the certain outcome is framed as a "gift certificate" while the lottery is framed as "lottery ticket" (or coin flip, gamble, raffle). Mislavsky and Simonsohn (2018) find the uncertainty effect when subjects perceive the certain outcome as a more natural transaction than the lottery. They interpret the lottery as a transaction that has an unexplained feature.[4] Outside the laboratory, the choice of dominated options has been documented for several markets, e.g., for telecommunication services (Genakos et al., 2023) or health insurance (Bhargava et al., 2017).

Despite this evidence, only few papers have examined the uncertainty effect in strategic settings so far. Dreyfuss et al. (2022) and Meisner and von Wangenheim (2023) explore whether expectations-based reference-dependent preferences can explain misrepresentations in deferred acceptance mechanisms (which are known to be strategy-proof). Both in exper-

[3]Further applications of expectations-based loss aversion include Heidhues and Kőszegi (2008), Karle and Peitz (2014), and Karle and Möller (2020) on imperfect competition; Rosato (2017) on sequential bargaining; Benkert (2025) on bilateral trade; Carbajal and Ely (2016) and Hahn et al. (2015) on monopolistic screening; Herweg et al. (2010) and Macera (2018) on principal-agent contracts; Lange and Ratan (2010), Dato et al. (2018), and Balzer and Rosato (2021) on auctions or tournaments; Dato et al. (2017) on strategic interaction in finite games; Daido and Murooka (2016) on team incentives; and Koch and Nafziger (2016) on mental accounting.

[4]In addition, Andreoni and Sprenger (2011) also find the uncertainty effect in their experimental data. Some studies demonstrate that the uncertainty effect does not show up under certain conditions; see Rydval et al. (2009) and Wang et al. (2013).

iments and in the field, there is a substantial share of individuals who chooses first-order stochastically dominated options.[5] Loss-averse individuals may employ such behavior in order to avoid disappointments. In the context of product switching, our companion paper (Karle et al., 2023) motivates the idea that the uncertainty effect can generate scale-dependent psychological switching costs. The present paper is the first that examines the interaction of attachment and uncertainty effect in a strategic setting.

Persuasion with behavioral receivers. More generally, we contribute to the literature that considers persuasion with boundedly rational senders or receivers, see, for example, Hagenbach and Koessler (2017) as well as Bilancini and Boncinelli (2018) for signaling, and Blume and Board (2013), Glazer and Rubinstein (2012, 2014), Galperti (2019), Giovannoni and Xiong (2019), Hagenbach and Koessler (2020), as well as Eliaz et al. (2021) for cheap talk. In contrast to these papers, we assume that agents have fully rational beliefs, while the receiver exhibits expectations-based loss-averse preferences. The main innovation of our model is that it allows for persuasion through signaling in a setting where the receiver is perfectly informed about the realization of all payoff-relevant variables when she chooses between options.

3 The Model

A loss-neutral sender interacts with a loss-averse receiver in two periods. In period 1, the sender makes an early offer (v^s, t^s) to the receiver, where $v^s \in [0, 1]$ is the regular value of the offer to the receiver and $t^s \in \mathbb{R}$ a (positive or negative) transfer from the sender to the receiver. In period 2, the receiver learns about her outside option $(v^o, 0)$, where $v^o \in [0, 1]$. She then chooses between the sender's offer (v^s, t^s) and her outside option $(v^o, 0)$. The distinction between regular value and transfer allows the sender to offer something that the outside option does not provide, e.g., through product differentiation.

If the receiver accepts the outside option, her consumption utility is v^o and the sender's payoff is zero. If the receiver accepts the sender's offer, her consumption utility is $v^s + t^s$ and the sender's payoff is $1 - v^s - t^s$. The shape of the sender's payoff function ensures that the sender can profitably trade with the receiver even if the receiver's outside option is maximal, and that regular value and transfer are fungible for the sender. The outside option value v^o is distributed according to the distribution function F with continuously differentiable density f. F has full support on the unit interval and the slope of density f is bounded. Let $\max f'$ be the maximum of the slope of density f on the unit interval. For several results we will require that f is weakly increasing on the unit interval. In period 1, the sender observes the realization of

[5] For example, they may rank a funded position (study place with a fellowship) below an identical nonfunded one; see Dreyfuss et al. (2022) for a summary of the empirical evidence.

v^o and can condition his offer on this value, while the receiver only knows the distribution of v^o. Figure 1 shows the timeline of the interaction between sender and receiver.

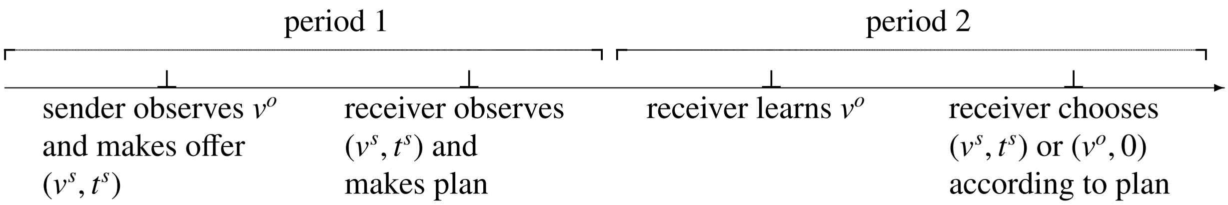

Figure 1: Timeline

Preferences. To model the receiver's expectations-based loss aversion we follow Kőszegi and Rabin (2006, 2007). Her payoff from accepting an option in period 2 consists of two components: consumption utility and gain-loss utility from comparisons of the actual outcome to a reference point. This reference point is defined by the receiver's period-1 expectations. Suppose that in period 1 she expects to accept the option $(\tilde{v}, \tilde{t})$ with certainty in period 2. If she indeed accepts option (v, t), her payoff equals

$$U(v, t \mid \tilde{v}, \tilde{t}) = v + t + \mu(v - \tilde{v}) + \mu(t - \tilde{t}). \tag{1}$$

The function μ captures gain-loss utility. We assume that μ is piecewise linear with slope η for gains and slope $\eta\lambda$ for losses; $\eta > 0$ is the weight of gain-loss utility relative to consumption utility, and $\lambda > 1$ is the receiver's degree of loss aversion.

The receiver may have stochastic expectations over outcomes. Let the distribution functions G^v and G^t denote her period-1 expectations regarding the outcome in the value and transfer dimension, respectively. The receiver's payoff from accepting option (v, t) is given by

$$U(v, t \mid G^v, G^t) = v + t + \int \mu(v - \tilde{v})\, \mathrm{d}G^v(\tilde{v}) + \int \mu(t - \tilde{t})\, \mathrm{d}G^t(\tilde{t}). \tag{2}$$

Thus, gains and losses are weighted by the probability with which the receiver expects them to occur. This preference model captures the following intuition. If the receiver expects to get either 0 or 1 in the value dimension, each with probability 50 percent, then an allocation of 0.6 feels like a gain of 0.6 weighted with 50 percent probability, and a loss of 0.4 also weighted with 50 percent probability.

Strategies and Equilibrium. The sender's strategy defines his offer (v^s, t^s) in period 1 based on the receiver's outside option value v^o. It is given by the measurable function[6]

$$\sigma^s : [0,1] \to [0,1] \times \mathbb{R}. \tag{3}$$

Thus, the sender's offer is potentially informative for the receiver about her outside option value v^o. Upon observing the sender's offer (v^s, t^s), the receiver updates her belief about her outside option value to $\hat{F} \equiv F(v^o \mid v^s, t^s)$. She then makes a plan under what circumstances she accepts which option. Formally, the receiver's plan is a strategy that defines her choice in period 2 between sender offer (v^s, t^s) and outside option $(v^o, 0)$ based on the features of these two options,

$$\sigma^r : [0,1] \times \mathbb{R} \times [0,1] \to [0,1] \times \mathbb{R}. \tag{4}$$

Given sender's strategy σ^s, his offer (v^s, t^s), and the receiver's strategy σ^r, we can define the receiver's expectations about period-2 outcomes. Let $G^v \equiv G^v(\tilde{v} \mid \sigma^s, \sigma^r, (v^s, t^s))$ denote her expectations about the outcome in the value dimension, and $G^t \equiv G^t(\tilde{t} \mid \sigma^s, \sigma^r, (v^s, t^s))$ her expectations regarding the outcome in the transfer dimension. For a given sender strategy σ^s, the receiver's strategy σ^r is a personal equilibrium (PE) if it is optimal for her in period 2 to always follow this plan. Moreover, strategy σ^r is a preferred personal equilibrium (PPE) if it is a PE and there is no alternative PE that yields her a higher expected payoff in period 1. An equilibrium of the game is given by a perfect Bayesian equilibrium where the receiver's strategy constitutes a preferred personal equilibrium. We state these definitions formally.

Definition 1. *For a given sender strategy σ^s, the receiver's strategy σ^r is a personal equilibrium (PE) if for any v^o and sender offer (v^s, t^s) we have*

$$U(\sigma^r(v^s, t^s, v^o) \mid G^v, G^t) \geq U(v, t \mid G^v, G^t)$$

at each available option $(v, t) \in \{(v^s, t^s), (v^o, 0)\}$. For a given sender strategy σ^s, the receiver's strategy σ^r is a preferred personal equilibrium (PPE) if it is a personal equilibrium and for any sender offer (v^s, t^s) we have

$$\mathbb{E}_{\hat{F}}[U(\sigma^r(v^s, t^s, v^o) \mid G^v, G^t)] \geq \mathbb{E}_{\hat{F}}[U(\tilde{\sigma}^r(v^s, t^s, v^o) \mid \tilde{G}^v, \tilde{G}^t)]$$

at any alternative personal equilibrium $\tilde{\sigma}^r$.

[6]In this paper, we restrict attention to pure strategies for tractability reasons. This assumption is not without loss of generality. For example, in Heidhues and Kőszegi (2014) model of sales, the monopolist benefits from committing to a non-trivial distribution over prices.

Definition 2. *The triple $\sigma = (\sigma^s, \sigma^r, \hat{F})$ is a perfect Bayesian equilibrium if, for any outside option value $v^o \in [0, 1]$, the sender's offer $\sigma^s(v^o)$ maximizes his expected payoff for given σ^r, strategy σ^r is a PPE for given σ^s, and $\hat{F}$ is derived from σ^s and Bayes' rule whenever possible.*

This model defines a signaling game in which the sender (potentially) signals his private information about the receiver's outside option through the early offer to the receiver. There is no restriction on out-of-equilibrium beliefs through a refinement like, for example, the Intuitive Criterion. Thus, the receiver may draw any conclusion about her outside option value from observing an out-of-equilibrium offer. Formally, this means that, for a given out-of-equilibrium offer, we can select any distribution with support on the unit interval as the receiver's prior about her outside option value. As tie-breaking rule we assume that the receiver accepts the sender's offer in period 2 if she is indifferent between the sender's offer and the outside option. To illustrate our model, we refine the two examples from the introduction.

Market Example (Continuation). Many providers make poaching offers to the consumer (receiver), so they compete in Bertrand manner. The consumer's current telecommunications service provider (sender) has a competitive advantage through superior information which translates into lower production costs. In contrast to the receiver, the sender knows all contract offers that are available to the receiver. The regular value is product quality (or the extent of product services) and the transfer is a price reduction (a negative transfer hence indicates a price increase). Consider the following parametrization: The outside option offered by other providers has regular value $\zeta + v^o$ and production costs $c^o = \zeta$ so that the competitive price equals $p^o = \zeta$. The sender's contract offer has regular value $\zeta + v^s$, production costs $c^s = \zeta + v^s - 1$, and price $p^s = \zeta - t^s$. If the receiver accepts his offer, the sender's payoff is $p^s - c^s = 1 - v^s - t^s$. This parametrization is equivalent to our setting for any value $\zeta \in \mathbb{R}$. We therefore can normalize $\zeta = 0$ without loss of generality.

Negotiation Example (Continuation). Both the HR representative (sender) and the young professional (receiver) prefer the current match to other arrangements. Therefore, they divide a pie of size 1 among themselves. The receiver's bargaining power equals v^0 (i.e., this would be the value that she would get if she does not sign a contract with the sender). The sender makes a take-it-or-leave-it offer, which may contain a feature that the outside option (with certainty) does not have. The sender knows which deal the receiver could get elsewhere if negotiations break down, while the receiver has to find out this information (in the next job interviews) after observing the sender's proposal. The regular value may capture non-wage job attributes (like flexibility or work autonomy), while the transfer captures the receiver's compensation.

4 Signaling Equilibria

We begin the equilibrium analysis with two definitions: The sender's offer (v^s, t^s) is called inferior at outside option value v^o if $v^s + t^s < v^o$. Further, we say that the sender benefits from making early offers if the receiver accepts an inferior option at any positive outside option value $v^o > 0$. In this section, we study under what circumstances there exists an equilibrium in which the sender benefits from making early offers. In Subsection 4.1, we first discuss the benchmark case when the receiver is loss neutral and then examine some preliminary results for a loss-averse receiver. In Subsection 4.2, we state the main result and explain the structure of signaling equilibria in which the sender persuades the receiver to accept an inferior offer at any positive outside option value. Finally, in Subsection 4.3, we examine the properties of sender-preferred equilibria.

4.1 Preliminaries

We consider first the benchmark case when the receiver is loss neutral, $\lambda = 1$. In this case, she is not bothered by gain-loss sensations and accepts a sender's offer (v^s, t^s) only if $v^s + t^s \geq v^o$. In equilibrium, the sender will then, for any $v^o < 1$, make an offer with $v^s + t^s = v^o$ so that his profit equals $1 - v^o$. Making early offers has no particular value for the sender in this setting and there is no scope for persuasion through signaling.

From now on we focus on the case when the receiver is loss averse, $\lambda > 1$. We obtain the following observation: An equilibrium in which the sender benefits from making early offers cannot be a pooling or a separating equilibrium. First, a pooling equilibrium does not exist: In a pooling equilibrium, the sender would make the same offer (v^s, t^s) at every outside option value. An offer (v^s, t^s) with positive total value $v^s + t^s > 0$ cannot be an equilibrium offer in a pooling equilibrium since at sufficiently low values of the outside option the sender would have an incentive to make a less generous offer. For example, if $v^o < v^s + t^s$, the sender would benefit from offering $(v^o + \varepsilon, 0)$ with $\varepsilon > 0$ and $v^o + \varepsilon < v^s + t^s$; in period 2, the receiver would strictly prefer $(v^o + \varepsilon, 0)$ to her outside option, regardless of her reference point. Further, an offer (v^s, t^s) with weakly negative total value $v^s + t^s \leq 0$ cannot be an equilibrium offer in a pooling equilibrium since the receiver would reject it in period 2 if her outside option value v^o is sufficiently close to 1 (in which case the sender would like to make another offer).[7] Therefore, an equilibrium in which the sender benefits from making early offers cannot be a pooling equilibrium.

Next, a separating equilibrium does exist. However, in any such equilibrium, there is no scope for exploiting the receiver's loss aversion through early offers. In a separating equilib-

[7]To see this note that $v^s \in [0, 1]$ and that we must have $t^s < 0$ if $v^s > 0$.

rium, the receiver infers her outside option value from the sender's offer. Suppose that at some value $v^o \in (0, 1)$ the sender's equilibrium offer is (v^s, t^s). The sender is willing to make this offer only if $v^s + t^s \leq v^o$ (if $v^s + t^s > v^o$, he could again deviate profitably by offering $(v^o + \varepsilon, 0)$ with $\varepsilon > 0$ and $v^o + \varepsilon < v^s + t^s$, which the receiver would accept in period 2). In period 1, the receiver is willing to plan the acceptance of the sender's offer only if $v^s + t^s \geq v^o$. Hence, we must have $v^s + t^s = v^o$. Assume that $t^s \geq 0$ (a similar argument applies for the case $t^s \leq 0$). In period 2, the receiver then indeed accepts the sender's offer if

$$v^s + t^s \geq v^o + \eta(v^o - v^s) - \eta\lambda t^s. \tag{5}$$

Since $\eta > 0$ and $\lambda > 1$, this inequality implies the following: If $t^s > 0$, the receiver would accept offer (v^s, t^s) in period 2 even if her outside option value is slightly larger than v^o to avoid the loss in the transfer dimension. The sender would then have an incentive to offer (v^s, t^s) at other outside option values as well. Thus, we must have $t^s = 0$ and $v^s = v^o$ at any outside option value $v^o \in [0, 1]$. This implies that the sender does not benefit from making early offers in a separating equilibrium.[8] An equilibrium in which the sender persuades the receiver to accept an inferior option at any $v^o > 0$ therefore must be semi-separating.

In a semi-separating equilibrium, the sender's offer can be informative about the receiver's outside option without revealing its exact value. Suppose that if the receiver gets the early offer (v^s, t^s), this informs her that her outside option is located in the non-empty set $V \subset [0, 1]$. Define $\underline{v} = \inf(V)$ and $\bar{v} = \sup(V)$; we will use this notation throughout the paper. The receiver's PE then must be a cut-off plan. The reason for this is that, at any given plan σ^r, the receiver's utility from accepting the offer (v^s, t^s) is constant, while her utility from accepting the outside option strictly increases in v^o. Hence, for any PE, there exists a value $v^* \in [0, 1]$ so that the receiver chooses the outside option if $v^o > v^*$ and accepts the sender's offer if $v^o \leq v^*$. In general, there can be multiple PEs and it could be cumbersome to determine the PPE. However, if f is weakly increasing on the unit interval, we obtain a result that substantially simplifies the analysis.

Lemma 1. *Let f be weakly increasing on the unit interval. Consider any sender strategy σ^s where for some non-empty set $V \subset [0, 1]$ the sender makes the offer (v^s, t^s) with $v^s + t^s \leq \underline{v}$ and $v^s \leq \underline{v}$ if and only if $v_o \in V$. Any cut-off plan σ^r that maximizes the receiver's expected payoff in period 1 at σ^s and (v^s, t^s) then specifies either (i) to always accept (v^s, t^s) when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.*

[8]To prove the existence of a separating equilibrium, we can use the following out-of-equilibrium beliefs: If the receiver observes an offer (v^s, t^s) with $t^s \neq 0$, she believes in period 1 that $v^o = 1$ with certainty. If $v^s + t^s < 1$, she plans to accept the outside option. It is then impossible for the sender to make an out-of-equilibrium offer that the receiver accepts and that offers less consumption value than the outside option.

Lemma 1 implies the following: To show that the plan[9] "accept (v^s, t^s) when $v_o \in V$" is a PPE after offer (v^s, t^s) is made, we only have to make sure that it is a PE and that it is weakly better for the receiver than the plan "accept the outside option when $v_o \in V$." In the proof of Lemma 1, we show that plans with intermediate cut-off levels $v^* \in (\underline{v}, \bar{v})$ do not maximize the receiver's expected payoff. Such plans generate gain-loss sensations in both the regular value and the transfer dimension. Since, by assumption, we have $v^s + t^s \leq \underline{v}$, they also do not generate more consumption utility than the plan "accept the outside option when $v_o \in V$." The assumption on the distribution F then ensures that either the certain rejection or the certain acceptance of the sender's offer (or both plans) maximize the receiver's expected payoff.

4.2 Main Result

When the sender makes an early offer (v^s, t^s) where the receiver knows that its total value $v^s + t^s$ is lower than that of any possible outside option, why should the receiver plan to accept it? For loss-averse receivers there is an important reason why planning to accept such an offer can be optimal. In period 1, she would then enjoy peace of mind as she will not be exposed to gain-loss sensations in period 2. Of course, accepting (v^s, t^s) must also be optimal in period 2, so the total value $v^s + t^s$ of the sender's offer cannot be too small relative to the outside option value v^o. In an equilibrium in which the sender persuades the receiver to accept an inferior offer at any $v^o > 0$, these forces must be balanced.

We show that there can exist an equilibrium where, at any positive outside option value $v^o > 0$, the sender makes (and the receiver accepts) an inferior offer (v^s, t^s). To state this result and to simplify the subsequent discussion, we refer to a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ and define $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$ for each interval V_i. Throughout this section, we assume that the sequence $\{V_i\}_{i\in\mathbb{N}}$ partitions the unit interval, and that intervals are descending, in the sense that $\bar{v}_{i+1} = \underline{v}_i$. We now can state our main result.

Proposition 1 (Signaling Equilibria)**.** *If $\eta(\lambda - 1) > 3(1 + 2\max f')$ and f is weakly increasing on the unit interval, an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o > 0$. Any such equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, which partition the unit interval, and values $\{w_i\}_{i\in\mathbb{N}}$ so that the sender makes an offer (v^s, t^s) with total value $v^s + t^s = w_i < \underline{v}_i$ and non-zero transfer t^s if $v^o \in V_i$; the receiver always accepts this offer.*

[9]This is not a fully specified strategy σ^r. Throughout the paper, we will use this "reduced" description of a strategy whenever it is not necessary to specify all details of the "complete" strategy.

The equilibrium suggested in Proposition 1 is a signaling equilibrium in which the receiver learns from an early offer about the interval in which her outside option value is located. It is shaped by three forces: the uncertainty effect, the attachment effect, and the sender's incentive to make offers that are as low as possible, but are still accepted by the receiver. We explain each force in detail and elaborate what it implies for the structure of the signaling equilibrium.

The Uncertainty Effect. Suppose the receiver gets an offer (v_i^s, t_i^s) that informs her that her outside option value is in the interval V_i, with $\underline{v}_i = \inf(V_i)$ and $\bar{v}_i = \sup(V_i)$. Assume that $v_i^s + t_i^s < \underline{v}_i$ and $v_i^s \leq \underline{v}_i$. By Lemma 1, the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" is a PPE if it is a PE and if in period 1 its expected payoff exceeds the expected payoff from the plan "accept the outside option when $v_o \in V_i$." The uncertainty effect implies that the latter requirement can be met even if accepting the outside option generates strictly more consumption utility than $v_i^s + t_i^s$. The reason is that the plan "accept the outside option when $v_o \in V_i$" has the potential for disappointments, that is, the realized outside option value may be close to the lower bound $\underline{v}_i$ in which case the receiver experiences a loss (relative to higher values of the outside option that were possible ex ante). Formally, the receiver weakly prefers the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" to "accept the outside option when $v_o \in V_i$" if[10]

$$v_i^s + t_i^s \geq \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{6}$$

Whether this inequality is satisfied or not depends on the distribution over possible outside option values $\hat{F}$. Given that $\hat{f}$ is weakly increasing on its support, the distribution $\hat{F}$ that minimizes the right-hand side of inequality (6) is the uniform distribution on the interval $(\underline{v}_i, \bar{v}_i)$. The intuition is that this distribution "maximizes" the uncertainty the receiver is exposed to. If $\hat{F}(v)$ is indeed a uniform distribution, then inequality (6) is satisfied if $\eta(\lambda - 1) > 3$ and $v_i^s + t_i^s$ is sufficiently close to $\underline{v}_i$.

Note that the first statement of Proposition 1 holds for all distributions with weakly increasing density f. Hence, we need a further element to ensure that the uncertainty effect unfolds for the full range of positive outside option values. As the interval $(\underline{v}_i, \bar{v}_i)$ becomes small, we can approximate the inequality in (6) by an expression that only depends on the loss aversion parameters η, λ and the maximum $\max f'$ of the slope of function f. Hence, the threshold $\eta(\lambda - 1) > 3(1 + 2 \max f')$ holds for all distributions F with weakly increasing density f since

[10] An intuitive way to understand the gain-loss term on the right-hand side of inequality (6) is as follows: Take two values $\tilde{v}, v \in (\underline{v}_i, \bar{v}_i)$ with $\tilde{v} > v$. With "probability" $\hat{f}(\tilde{v})$ (resp. $\hat{f}(v)$) the outcome in the value dimension is $\tilde{v}$ (resp. v) and with the same "probability" the reference-point in the value dimension is $\tilde{v}$ (resp. v). Hence, with "probability" $\hat{f}(v)\hat{f}(\tilde{v})$ the outcome is $\tilde{v}$, while the reference point equals v, so that the receiver experiences a gain of $\eta(\tilde{v} - v)$; with the same "probability" outcome and reference point are reversed, so that the receiver experiences a loss of $\eta\lambda(\tilde{v} - v)$. The net effect is therefore $-\eta(\lambda - 1)(\tilde{v} - v)$.

we can always choose the intervals in $\{V_i\}_{i\in\mathbb{N}}$ small enough such that inequality (6) is satisfied for some offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $v_i^s \leq \underline{v}_i$.

The Attachment Effect. We consider the situation where the receiver gets an offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ that informs her that her outside option value is in the interval V_i. The receiver follows the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" only if this plan is a PE. For this, it must be optimal for the receiver to accept the sender's offer in period 2 even if the outside option value equals $\bar{v}_i$. Given the expectations induced by the plan "accept (v_i^s, t_i^s) when $v_o \in V_i$" this is the case if and only if

$$v_i^s + t_i^s \geq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{7}$$

If the inequalities in (6) and (7) are satisfied, then the receiver's PPE specifies to accept (v_i^s, t_i^s) at all outside option values $v^o \in V_i$. From inequality (7) we can make two important observations. First, the payoff-maximizing way for the sender to make an offer that satisfies inequality (7) is to create the total value only through the transfer t_i^s. Accepting the outside option implies losing the transfer, which through loss aversion is particularly painful for the receiver; we can observe this from the term $\eta\lambda t_i^s$. As a result, the receiver is "attached" to the offer. Second, and relatedly, inequality (7) puts an upper bound on the length of the interval V_i. If the total value is smaller than the lowest possible outside option value, $v_i^s + t_i^s < \underline{v}_i$, then both inequalities taken together imply that we must have $\underline{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$.

Sender Incentives. The proposed equilibrium is a signaling equilibrium only if the sender has an incentive to make the offer (v_i^s, t_i^s) if and only if $v^o \in V_i$. Specifically, he must not have an incentive to make this offer when $v^o > \bar{v}_i$. Offers and intervals therefore must be chosen such that the receiver rejects an offer (v_i^s, t_i^s) if her true outside option (unexpectedly) exceeds $\bar{v}_i$. Note that upon receiving offer (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ the receiver expects that $v^o \in V_i$ and that she accepts this offer in period 2. It is then optimal for her to reject it for any $v^o > \bar{v}_i$ if and only if

$$v_i^s + t_i^s \leq \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{8}$$

The right-hand side of this inequality is the expected payoff from accepting the outside option when $v^o = \bar{v}_i$. Since the inequality in (7) also needs to be satisfied to ensure sufficient attachment, the offer (v_i^s, t_i^s) for an interval V_i must be chosen such that

$$v_i^s + t_i^s = \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{9}$$

For given v_i^s and interval V_i the transfer t_i^s must therefore satisfy $t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{1+\eta}{1+\eta\lambda}v_i^s$. Thus, the sender can reduce the total value $v_i^s + t_i^s$ while respecting (9) by substituting transfer t_i^s for regular value v_i^s. The scope for this substitution may be constrained by the fact that it still must be optimal for the receiver to plan acceptance in period 1. Formally, this means that the inequality in (6) must be satisfied as well. Observe that the smallest possible total value that the sender can offer to the receiver according to the condition in (9) is $v_i^s + t_i^s = \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$. However, if $\underline{v}_i$ is relatively close to $\bar{v}_i$, then according to the condition in (6) the total value $v_i^s + t_i^s$ must be relatively close to $\bar{v}_i$. Since $\bar{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$, it may be necessary to make offers with both positive regular value v_i^s and positive transfer t_i^s to satisfy both the condition in (6) and the condition in (9): The transfer t_i^s is then positive to exploit the attachment effect, and the value v_i^s is positive to enable credible signaling.

The uncertainty effect, the attachment effect, and the sender's incentives constrain the intervals $\{V_i\}_{i\in\mathbb{N}}$ and the offers $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ of an equilibrium in which the sender persuades the receiver to accept an inferior offer at all positive outside option values. To construct such an equilibrium, one also has to fix out-of-equilibrium beliefs. It must be optimal for the receiver to reject any out-of-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$ when $\tilde{v}_i^s + \tilde{t}_i^s < v^o$. One option is to assume "optimistic beliefs", that is, the receiver believes in period 1 that her outside option value is maximal, $v^o = 1$, after receiving an out-of-equilibrium offer $(\tilde{v}_i^s, \tilde{t}_i^s)$.[11] Given this belief, it is indeed optimal for the receiver to plan the rejection of $(\tilde{v}_i^s, \tilde{t}_i^s)$ and eventually reject this offer if $\tilde{v}_i^s + \tilde{t}_i^s < v^o$.

4.3 Sender-Preferred Equilibria

Like in most signaling games, there are many equilibria in our setting. These include an equilibrium in which the sender's offer matches the receiver's outside option for any value $v^o \in [0, 1]$; see the discussion of the existence of a separating equilibrium at the beginning of Section 4. Classic refinements like the Intuitive Criterion (Cho and Kreps, 1987) or Undefeated Equilibrium (Mailath et al., 1993) do not reduce the number of equilibria in a meaningful way in our case. The reason is that there can be a continuum of offers where each offer is optimal at a certain outside option value given that the receiver plans their acceptance in period 1. In order to select between equilibria, we examine equilibria in which the sender earns the highest possible ex ante expected payoff, that is, the "sender-preferred equilibrium."[12]

A sender-preferred equilibrium is the solution to a complex optimization problem. Suppose that a sender-preferred equilibrium has an interval structure as suggested in Proposition 1. The length of the intervals $\{V_i\}_{i\in\mathbb{N}}$ determines the strength of the uncertainty effect and hence the maximal difference between the highest possible outside option value $\bar{v}_i$ and the total value

[11] This is the analog to "pessimistic beliefs" which are frequently assumed in job-market signaling models in order to motivate the equilibrium strategies.

[12] Alonso and Câmara (2018) use a similar solution concept. In this subsection, we make the additional technical assumption that in an equilibrium σ it must be the case that the strategies σ^s and σ^r are such that the seller's ex ante payoff is well-defined.

of the sender's offer $v_i^s + t_i^s$. This maximal difference increases in the length of V_i. However, reducing the length of an interval V_i by increasing its lower bound $\underline{v}_i$ has the advantage that receiver types around $\underline{v}_i$ would obtain a lower value since $v_{i+1}^s + t_{i+1}^s < v_i^s + t_i^s$. Hence, the optimal configuration of intervals depends on the local properties of the distribution F. Additionally, all offers (v_i^s, t_i^s) need to be chosen such that the receiver indeed accepts them in period 1.

We can show that the sender-preferred equilibrium must have two properties. First, it must have an interval structure as indicated in Proposition 1 in order to generate an uncertainty effect. Second, the sender's offers must contain non-negative transfers to exploit the attachment effect. We therefore obtain the following result:

Proposition 2 (Sender-Preferred Equilibrium)**.** *If $\eta(\lambda - 1) > 3(1 + 2 \max f')$ and f is weakly increasing on the unit interval, then any sender-preferred equilibrium is characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$, which partition the unit interval, and values $\{w_i\}_{i\in\mathbb{N}}$ so that the sender makes an offer (v^s, t^s) with total value $v^s + t^s = w_i \leq \underline{v}_i$ if $v^o \in V_i$; the transfer t^s is non-zero if $\underline{v}_i < v^o$; the receiver accepts this offer if $v^o < 1$.*

The intuition behind Proposition 2 is that there cannot exist a sender-preferred equilibrium σ in which the sender makes offers of differing total value for any two outside option values from an interval $V = (v_L, v_H) \subset [0, 1]$. The receiver would then be able to infer her outside option value from these offers so that the total value of a sender offer would have to be equal to the outside option value for each $v^o \in V$. In this case, we can construct an alternative equilibrium σ' which dominates σ in terms of expected payoff for the sender. We would make the same offer for all values of a subinterval of V and the total value of this offer would be strictly smaller than the average outside option value of the subinterval. Therefore, any sender-preferred equilibrium must have an interval structure as suggested in Proposition 1.

We can say even a bit more about the sender-preferred equilibrium than stated in Proposition 2. For outside options in the highest interval $(\underline{v}_1, \bar{v}_1] = (\underline{v}_1, 1]$ it must be the case that the sender makes an offer with strictly positive transfer. In this interval, he cannot differentiate his offer from the outside option through a large regular value and negative transfer. To see this, suppose that for an outside option value v^o in this interval the sender makes the offer (v^s, t^s) with $t^s < 0$. If v^o is sufficiently close to 1, we then have

$$v^s + t^s \leq 1 - |t^s| < v^o - \eta\lambda(1 - v^o) + \eta\,|t^s|, \tag{10}$$

where the right-hand side of this inequality is a lower bound on the receiver's payoff from accepting the outside option in period 2 after planning to accept the sender's offer in period 1. Hence, the receiver would reject the sender's offer if the outside option value is sufficiently large. Offer (v^s, t^s) would not have been made in equilibrium at outside option value v^o.

5 Uncertainty in Multiple Dimensions

In our baseline model, the sender benefits from making early offers if the receiver's loss aversion parameters η, λ are sufficiently large. We at least need $\eta(\lambda - 1) > 3$. The reason for this requirement is that there is uncertainty only in one outcome dimension (i.e., the regular value dimension). This has the following consequence: If for a given upper bound of the potential outside option values $\bar{v}$ the sender wants to make a more generous offer – that is, move the total value $v^s + t^s$ closer to $\bar{v}$ – this restricts the possible values of $\underline{v}$ and hence reduces in a signaling equilibrium the uncertainty about the outside option value and the strength of the uncertainty effect. As $v^s + t^s$ approaches $\bar{v}$, the uncertainty effect vanishes. Therefore, if there is uncertainty only in one outcome dimension, then the attractiveness of an offer relative to the outside option and the extent of the uncertainty effect are tightly linked.

Degrees of loss aversion that satisfy $\eta(\lambda - 1) > 3$ are empirically relevant[13] and the uncertainty effect has been found in numerous settings (as discussed in Section 2). However, in theoretical applications of expectations-based loss-averse preferences, the assumed levels of loss aversion are typically smaller. In this section, we present an extension of the model in which our main results – persuasion through signaling as well as interaction of attachment and uncertainty effect – obtain for all loss aversion parameters η, λ that satisfy $\eta(\lambda - 1) > 0$. The idea behind this extension is that there is uncertainty about the outside option in multiple outcome dimensions that are payoff relevant for the receiver. To motivate it, we again consider our running examples.

Market Example (Continuation). If the outside option is a utilities contract, the receiver may face uncertainty about contract specifications, delivery times, customer support, warranties, payment details, online registry details, and so forth. In contrast, she observes these details immediately for the current provider's offer.

Negotiation Example (Continuation). As long as the young professional has not completed all job interviews, she is uncertain about many aspects of the best alternative offer: commuting times, home office regulation, promotion opportunities, the need to spend time on business trips. The HR representative who conducts the first interview clarifies these issues for his firm.

[13] See, for example, von Gaudecker et al. (2011) and Brown et al. (2024).

The receiver may not evaluate the joint value of these different attributes, but narrowly brackets them so that gain-loss sensations occur in multiple dimensions. As a result, some uncertainty about the specification of the outside option remains even if the receiver knows the exact value of the outside option v^o. In a signaling equilibrium, this relaxes the link between the attractiveness of an offer and the strength of the uncertainty effect.

In the following, we extend our baseline model by assuming that the receiver faces uncertainty in multiple dimensions. The expectations-based loss-aversion framework of Kőszegi and Rabin (2006) explicitly allows for such a setting. In Subsection 5.1, we describe the updated version of our model. In Subsection 5.2, we show that, in this version of the model, the sender may benefit from making early offers, regardless of the loss aversion parameters as long as $\eta(\lambda - 1) > 0$. Finally, in Subsection 5.3, we discuss the features of sender-preferred equilibria in this setting.

5.1 Updated Setting

We consider the same model as in Section 3, with the difference that any option has values in two extra-dimensions, the x-dimension and the y-dimension. The sender's offer is now given by (v^s, t^s, x^s, y^s) and the outside option equals $(v^o, 0, x^o, y^o)$. The values in the x- and the y-dimension can be interpreted as design choices that involve trade-offs. For example, a utilities contract that comes with better customer service may also involve more advertising and unwanted email messages; a job that has more interesting business traveling is at times also more stressful. We therefore set $x^s + y^s = 0$ and $x^o + y^o = 0$. The sender chooses the regular value v^s, the transfer t^s, and the design $\xi^s \in \mathbb{R}$ of his early offer, where $x^s = \xi^s$ and $y^s = -\xi^s$.[14] We thus can abbreviate the sender's offer as (v^s, t^s, ξ^s).

For the outside option we have that v^o is distributed according to F on the unit interval, while the outcome in the transfer dimension is zero. The receiver faces uncertainty in the x- and y-dimension of the outside option: With probability $\frac{1}{2}$ we have $x^o = \xi^o + \xi$ and $y^o = -\xi^o - \xi$ for some values $\xi^o, \xi \in \mathbb{R}_+$ (state 1), and with probability $\frac{1}{2}$ we have $x^o = \xi^o - \xi$ and $y^o = -\xi^o + \xi$ (state 2). In period 1, the receiver knows the values ξ^o and ξ, but not the state. The parameter ξ thus captures the level of uncertainty in the extra-dimensions the receiver is exposed to if she plans to accept the outside option.

The consumption utility from any option is $v + t + x + y$. Hence, as in the baseline model, the consumption utility from the sender's offer equals $v^s + t^s$ and the consumption utility from the outside option equals v^o. If in period 1 the consumer expects to accept an offer $(\tilde{v}, \tilde{t}, \tilde{x}, \tilde{y})$

[14]We assume a trade-off in the extra-dimensions in order to keep the model (and the characterization of the receiver's PPE) tractable. Our normalization implies that both the sender's offer and the outside option provide the same total value in the extra-dimensions.

with certainty, and ends up choosing option (v, t, x, y), her utility equals

$$U(v, t, x, y \mid \tilde{v}, \tilde{t}, \tilde{x}, \tilde{y}) = v + t + x + y + \mu(v - \tilde{v}) + \mu(t - \tilde{t}) + \mu(x - \tilde{x}) + \mu(y - \tilde{y}). \tag{11}$$

Therefore, the consumer may experience gain-loss sensations in four dimensions (instead of two). The sender's strategy defines his offer in period 1 based on the receiver's outside option value v^o. It is given by the measurable function[15]

$$\sigma^s : [0, 1] \to [0, 1] \times \mathbb{R}^3. \tag{12}$$

Upon observing the sender's offer (v^s, t^s, ξ^s), the receiver updates her belief about her outside option value to $\hat{F} \equiv F(v^o \mid v^s, t^s, \xi^s)$. Her strategy defines her choice in period 2 between sender offer (v^s, t^s, x^s, y^s) and outside option $(v^o, 0, x^o, y^o)$ based on the features of these two options,

$$\sigma^r : [0, 1] \times \mathbb{R}^3 \times [0, 1] \times \mathbb{R}^2 \to [0, 1] \times \mathbb{R}^3. \tag{13}$$

Given sender's strategy σ^s, his offer (v^s, t^s, ξ^s), and the receiver's strategy σ^r, we can define the receiver's expectations about period-2 outcomes. The rest of the model proceeds as before. For $\xi^s = \xi^o$ and $\xi = 0$ the new version of the model would be equivalent to the baseline model.

5.2 Signaling Equilibria

We first adapt Lemma 1 to the new environment. Again, the receiver's PE must be a cut-off plan. This plan can be contingent on the state, i.e., the receiver may adopt different cut-off levels in the two states. Depending on the sender's design choice ξ^s, a state-contingent plan may maximize the receiver's expected payoff in period 1. To see this, note that the plan "accept the outside option in state 1 when $v_o \in V$ (for some interval V) and accept the sender's offer (v^s, t^s, ξ^s) in state 2 when $v_o \in V$" does not generate any gain-loss sensations in the extra-dimensions if $\xi^s = \xi^o + \xi$. In both states, the outcome in the x-dimension would be $\xi^o + \xi$ and the outcome in the y-dimension would be $-\xi^o - \xi$. Such a plan may maximize the expected payoff of the receiver in period 1. Nevertheless, if the sender chooses the design $\xi^s = \xi^o$, then the receiver cannot increase her expected payoff by adopting a plan with state-contingent cut-off levels. In this case, a cut-off plan that maximizes her expected payoff in period 1 is

[15] The assumption that the sender can condition his offer only on the outside option value implies that he cannot make state-dependent offers. One possible interpretation for this is that he does not observe the realization of the state at the point in time when he makes the offer to the sender. We make this assumption to keep the model tractable. However, we conjecture that allowing for state-dependent offers would not change our main results. Making them would inform the receiver about the state, which would reduce her uncertainty about the outside option and hence the strength of the uncertainty effect.

either "always accept the sender's offer when $v_o \in V$" or "always accept the outside option when $v_o \in V$", as in Lemma 1. This result is independent of the level of uncertainty ξ in the extra-dimensions.

Lemma 2. *Consider the model with uncertainty in the extra-dimensions. Let f be weakly increasing on the unit interval. Consider any sender strategy σ^s where for some interval $V \subset [0,1]$ the sender makes the offer (v^s, t^s, ξ^s) with $v^s + t^s \leq \underline{v}$ and $v^s \leq \underline{v}$ if and only if $v_o \in V$.*

(a) Any cut-off plan σ^r that maximizes the receiver's expected payoff in period 1 at σ^s and (v^s, t^s, ξ^s) then specifies either (i) to always accept (v^s, t^s, ξ^s) when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$, or (iii) to accept the outside option in state 1 when $v_o \in V$ and to accept (v^s, t^s, ξ^s) in state 2 when $v_o \in V$ (or vice versa).

(b) If $\xi^s = \xi^o$, then any cut-off plan σ^r that maximizes the receiver's expected payoff in period 1 at σ^s and (v^s, t^s, ξ^s) specifies either (i) to always accept (v^s, t^s, ξ^s) when $v_o \in V$, or (ii) to always accept the outside option when $v_o \in V$.

Using Lemma 2, we can characterize under what circumstances an equilibrium exists in which the sender persuades the receiver to accept an inferior offer at any outside option value. If the receiver plans to accept the outside option with certainty, she is exposed to additional gain-loss sensations through the uncertainty in the extra-dimensions. This increases the scope for the uncertainty effect. It is therefore conceivable that the critical threshold for the parameter $\eta(\lambda - 1)$ decreases as the uncertainty parameter ξ increases. However, we obtain a much stronger result. As long as $\eta(\lambda - 1) > 0$, there exists an equilibrium in which the sender benefits from making early offers, regardless of the level of uncertainty ξ in the extra-dimensions.

Proposition 3 (Signaling Equilibria, Uncertainty in Multiple Dimensions)**.** *Consider the model with uncertainty in extra-dimensions. If $\eta(\lambda - 1) > 0$ and f is weakly increasing on the unit interval, an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o \in [0,1]$.*

In the proof of Proposition 3, we construct the desired equilibrium through a finite sequence of disjoint intervals $\{V_i\}_{i=1}^n$ with $\bar{v}_{i+1} = \underline{v}_i$ for all $i = 1, ..., n-1$, so that the sender makes the offer (v_i^s, t_i^s, ξ_i^s) with total value $v_i^s + t_i^s < \underline{v}_i$ and $\xi_i^s = \xi^o$ if $v^o \in V_i$. This sequence partitions the interval $[\underline{v}, 1]$ for some value $\underline{v} \in (0,1)$. For any given outside option value $v^o \in [0, \underline{v})$, the sender makes an offer (v^s, t^s, ξ^s) with total value $v^s + t^s < 0$ and $\xi^s = \xi^o$. The sender chooses the design $\xi_i^s = \xi^o$ for each $i = 1, ..., n$ so that the receiver is – in each state – indifferent between accepting and rejecting the sender's offer in period 2 if v^o equals the upper bound $\bar{v}_i$. Hence, she would reject the offer if $v^o > \bar{v}_i$. Thus, the sender cannot gain from making an offer with total value below $v_i^s + t_i^s$ if $v^o \in V_i$.

There are two important new elements here. First, as we reduce the length of an interval $\bar{v}_i - \underline{v}_i$, the magnitude of expected gain-loss sensations (under the plan "always accept the outside option when $v_o \in V_i$") converges against a positive value, and not against zero as in the baseline model. This effect is due to the uncertainty in the extra-dimensions. Hence, for any given loss aversion parameters η, λ that satisfy $\eta(\lambda - 1) > 0$, if the interval V_i is short enough, we can find an offer (v_i^s, t_i^s, ξ_i^s) with $v_i^s + t_i^s < \underline{v}_i$ so that the receiver's expected payoff in period 1 is maximal if she plans to accept this offer as long as $v^o \in V_i$. The second new element is that an equilibrium in which the sender persuades the receiver through signaling to accept an inferior option at each outside option value does not necessarily have an interval structure. One can find an equilibrium in which the receiver always learns the value of her outside option v^o from the sender's offer, but nevertheless accepts an offer with a total value below v^o. The reason is that the plan "accept the outside option with certainty" always creates gain-loss sensations through the extra-dimensions, which the sender can exploit to convince the receiver to accept an inferior option.

5.3 Sender-Preferred Equilibria

A crucial difference between Proposition 1 and Proposition 3 is that – with uncertainty in the two extra-dimensions – an equilibrium in which the sender persuades the receiver to accept an inferior option does not necessarily have an interval structure. It can be a separating equilibrium. In order to say more about the features of an equilibrium with persuasion, we again focus on sender-preferred equilibria. We can show that if the uncertainty parameter ξ is small enough for given η, λ, then a sender-preferred equilibrium exhibits bunching of outside option values, as in our baseline model. The next result states this finding formally.

Proposition 4 (Sender-Preferred Equilibrium, Uncertainty in Multiple Dimensions)**.** *Consider the model with uncertainty in the extra-dimensions and suppose that f is weakly increasing on the unit interval. If for given values η, λ the parameter ξ is sufficiently small, then in a sender-preferred equilibrium the sender offers the same total value $v^s + t^s$ for all $v^o \in V$, where V is a subset of the unit interval that contains at least two elements.*

Proposition 4 indicates that a sender-preferred equilibrium σ cannot be a separating equilibrium if the uncertainty parameter ξ is sufficiently small for given loss aversion parameters η, λ. The intuition for this result is as follows: There are (potentially) two sources of uncertainty that contribute to the uncertainty effect – uncertainty about the outside option value and uncertainty in the extra-dimensions. When ξ is small, the latter source of uncertainty is relatively infertile. Starting from a separating equilibrium σ, the sender can benefit from creating additional uncertainty by offering the same total value $v^s + t^s$ for all outside option values in an interval $V \subset [0,1]$.

In this interval, the sender does not necessarily have to offer a non-zero transfer in order to attach the receiver to his offer (v^s, t^s, ξ^s). He can also choose a design ξ^s that is differentiated to the design of the outside option ξ^o so that (unexpectedly) choosing the outside option in period 2 creates a large loss-sensation in the extra-dimensions and hence is unattractive for the receiver. Thus, if the sender-preferred equilibrium exhibits bunching at outside option values, the sender can use product differentiation – sufficiently large or sufficiently small values of ξ^s – to ensure that the attachment effect is strong enough so that the receiver indeed accepts an inferior offer in period 2.

In a sender-preferred equilibrium, the sender persuades the receiver through signaling to accept an inferior option. This holds even if the receiver's degree of loss aversion and the uncertainty in the extra-dimension is small. Proposition 4 shows that, in this case, the sender-preferred equilibrium exhibits bunching of outside option values, as in the baseline model.

6 Extensions

We consider several extensions of our baseline model. In Subsection 6.1, we state a version of our main result that holds for general distributions of outside option values. In Subsection 6.2, we consider the case when, with some positive probability, the sender does not know the receiver's outside option value. Finally, in Subsection 6.3, we discuss the welfare consequences of persuasion through early offers in our setting.

6.1 General Distributions of Outside Option Values

Our main result holds under the assumption that f is weakly increasing on the unit interval. This assumption greatly facilitates the characterization of the receiver's PPE if the sender makes the same offer for all outside option values in a given interval. However, it rules out many distributions and thus may be seen as restrictive.

In the following, we relax this assumption and state a more general version of Proposition 1. Denote by $Z_j \subset [0,1]$ an interval with $\underline{z}_j = \inf(Z_j)$ and $\bar{z}_j = \sup(Z_j)$. Let $Z = \{Z_j\}_{j \in \mathbb{N}}$ be a finite or infinite sequence of such intervals with the property that $\bar{z}_{j+1} \leq \underline{z}_j$ for each $j \in \mathbb{N}$. We again assume that F has full support on the unit interval and that its density f is continuously differentiable. Define by $\max f'_Z$ the supremum of the slope of density f on the intervals in Z. We suppose that $\max f'_Z < \infty$. With these definitions we can state the following result:

Corollary 1 (General Distributions)**.** *If there is a sequence of intervals $Z = \{Z_j\}_{j\in\mathbb{N}}$ such that (i) we have $\eta(\lambda - 1) > 3(1 + 2\max f'_Z)$ and (ii) density f is weakly increasing on each interval Z_j, then an equilibrium exists in which the sender persuades the receiver through signaling to accept an inferior offer at each outside option value $v^o \in (\underline{z}_j, \bar{z}_j]$ with $j \in \mathbb{N}$.*

We obtain this result since we can find for each interval $(\underline{z}_j, \bar{z}_j]$ a sequence $\{V_i\}_{i\in\mathbb{N}}$ that partitions this interval as well as an equilibrium in which the sender makes an offer (v_i^s, t_i^s) with total value $v_i^s + t_i^s \in [\underline{z}_i, \underline{v}_i)$ if $v^o \in V_i$ and the receiver always accepts this offer. For this equilibrium, we assume that the sender offers $(v^s, t^s) = (v^o, 0)$ at any outside option value $v^o \notin (\underline{z}_j, \bar{z}_j]$, $j \in \mathbb{N}$. By construction, the receiver then accepts all equilibrium offers of the sender and (assuming optimistic beliefs) the sender cannot deviate profitably.

Corollary 1 shows that the sender may strictly benefit from making early offers (on a subset of the unit interval) even when f is not weakly increasing on its support. The sender then persuades the receiver through signaling to accept an inferior outside option for a subset of outside option values. Note that Corollary 1 allows for much more general distributions than Proposition 1, in particular distributions with more probability weight on small or intermediate outside option values than on large outside option values.

6.2 Uncertain Outside Option Value

In our baseline model, the sender always knows the sender's outside option value. One may ask to what extent the sender can benefit from making early offers if this assumption is not satisfied. Using the analysis from Section 4, we can address this question in the following simple extension. Suppose that with probability $\beta \in (0, 1)$ the sender knows the receiver's outside option value v^o in period 1 and with probability $1 - \beta$ he only knows the prior distribution over outside option values F. All other aspects of the model remain unchanged.

We examine under what circumstances an equilibrium exists in which the sender benefits from making early offers. For the case when the sender does not observe the receiver's outside option, we say that the sender persuades the receiver through signaling to accept an offer that is inferior to her outside option *in expectation* if the receiver accepts with certainty an offer $(\hat{v}^s, \hat{t}^s)$ that has less total value than the outside option in expectation,

$$\hat{v}^s + \hat{t}^s < \int_0^1 f(v) v \, \mathrm{d}v. \tag{14}$$

With this definition, we obtain the following result:

Corollary 2 (Uncertain Outside Option Value)**.** *Consider the model with uncertainty about the outside option value. If $\eta(\lambda-1) > \max\{1+\eta, 3(1+2\max f')\}$ and f is weakly increasing on the unit interval, an equilibrium exists in which the sender persuades the receiver through signaling to accept (i) an inferior offer at each outside option value $v^o > 0$ when the sender knows v^o and (ii) an offer that is inferior to her outside option in expectation when the sender does not know v^o.*

We show how to get this result. Assume that the sender makes the offer $(\hat{v}^s, \hat{t}^s)$ with $\hat{t}^s > 0$ if and only if he does not know the outside option value. If the sender gets the offer $(\hat{v}^s, \hat{t}^s)$, she infers that the sender does not know v^o and her prior about the outside option value distribution is given by F. In period 1, she then weakly prefers the plan "accept $(\hat{v}^s, \hat{t}^s)$ when $v^o \in [0,1]$" to the plan "accept the outside option when $v^o \in [0,1]$" if

$$\hat{v}^s + \hat{t}^s \geq \int_0^1 f(v)v\,\mathrm{d}v - \eta(\lambda-1)\int_0^1 f(v)\int_v^1 f(\tilde{v})(\tilde{v}-v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v. \tag{15}$$

Note that there is a range of total values $\hat{v}^s + \hat{t}^s$ that satisfy both inequality (14) and (15). Next, in period 2, the receiver is indeed willing to accept $(\hat{v}^s, \hat{t}^s)$ even at the highest possible outside option value $v^o = 1$ if and only if

$$\hat{v}^s + \hat{t}^s \geq 1 + \eta(1-\hat{v}^s) - \eta\lambda\hat{t}^s. \tag{16}$$

If the loss aversion parameter λ were small enough, the inequalities (14) and (16) would contradict each other. However, there is an open set of offers $(\hat{v}^s, \hat{t}^s)$ that satisfy all three inequalities (14) to (16) if f is weakly increasing on the unit interval and $\eta(\lambda-1) > 1+\eta$.[16]

We can now establish the desired result. Assume that f is weakly increasing on the unit interval and $\eta(\lambda-1) > \max\{1+\eta, 3(1+2\max f')\}$. Suppose the sender offers $(\hat{v}^s, \hat{t}^s)$ if he does not know v^o and he offers (v_i^s, t_i^s) if he knows v^o and $v^o \in V_i$. The arguments above and the analysis in Section 4 imply that these offers can be chosen such that (i) offer $(\hat{v}^s, \hat{t}^s)$ satisfies the inequalities (14) to (16) and it is different from any offer (v_i^s, t_i^s) for $i \in \mathbb{N}$, (ii) given the sender's strategy, it is a PPE for the receiver to accept each offer[17] unless it turns out that $v^o > \bar{v}_i$ after offer (v_i^s, t_i^s) has been made, and (iii) we have $v_i^s + t_i^s < \underline{v}_i$ for each $i \in \mathbb{N}$. For out-of-equilibrium offers we again assume optimistic beliefs so that such offers are unprofitable for the sender. We then get that the sender can benefit from making early offers (in expectation) even when

[16]To see this, note that (16) is satisfied with equality if we choose $\hat{v}^s = 0$ and $\hat{t}^s = \frac{1+\eta}{1+\eta\lambda}$. If f is weakly increasing on the unit interval, we have $\int_0^1 f(v)v\,\mathrm{d}v \geq \frac{1}{2}$. Further, we have that $\frac{1}{2} > \frac{1+\eta}{1+\eta\lambda}$ is equivalent to $\eta(\lambda-1) > 1+\eta$.

[17]Specifically, we can use arguments very similar to those used in the proof of Lemma 1 to show that the plan "accept $(\hat{v}^s, \hat{t}^s)$ when $v^o \in [0,1]$" characterizes the receiver's PPE if inequality (15) holds (given that the sender makes this offer if and only if he has no information about the outside option).

there is no asymmetric information between the sender and the receiver.

We have ruled out the case where the sender does not know the receiver's outside option value for sure ($\beta = 0$). This case is more complex since we can then no longer assume optimistic beliefs for out-of-equilibrium offers. As a consequence, we would have to determine the receiver's PPE for every possible offer. Many of these may be cut-off strategies with interior cut-off points – note that Lemma 1 does not apply to offers with total value above the outside option value. We leave this analysis for future research.

6.3 Welfare

We briefly comment on the effect of persuasion through early offers on the individual welfare of the sender and the receiver, respectively, as well as on aggregate welfare. So far, welfare statements are not common in the applied literature on expectations-based loss-averse preferences. The reason for this is that it is typically not clear to what extent gain-loss utility should be treated as part of normative preferences. We follow Goldin and Reck (2022) as well as Reck and Seibold (2023) and introduce a parameter $\pi \in [0, 1]$ that captures a social planner's judgment about the normative weight of gain-loss utility. The receiver's welfare in period 2 if she expected to accept option $(\tilde{v}, \tilde{t})$ with certainty and ends up accepting option (v, t) equals

$$U^*(v, t \mid \tilde{v}, \tilde{t}) = v + t + \pi\mu(v - \tilde{v}) + \pi\mu(t - \tilde{t}). \tag{17}$$

Hence, for $\pi = 0$ gain-loss utility is ignored for welfare judgments, while for $\pi = 1$ they receive the same normative weight as consumption utility. The sender's welfare G^* just equals his payoff. For aggregate welfare we use a simple utilitarian welfare function and add up the sender's and receiver's welfare, $G^* + U^*$.

To evaluate the welfare impact of persuasion through early offers, we first determine the equilibrium outcome in the absence of early offers. Suppose the sender can only make offers in period 2 when the receiver also knows her outside option value. As benchmark equilibrium we use the equilibrium in which the sender just matches the receiver's outside option value: At any outside option value v^o, he offers $(v^s, t^s) = (v^o, 0)$ in period 2 and the receiver accepts the sender's offer. The expected welfare of the receiver in this equilibrium equals

$$U_0^* = \int_0^1 f(v)v \, \mathrm{d}v - \pi\eta(\lambda - 1) \int_0^1 f(v) \int_v^1 f(\tilde{v}) (\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v, \tag{18}$$

while the expected welfare of the sender is given by

$$G_0^* = \int_0^1 f(v)(1 - v) \, \mathrm{d}v. \tag{19}$$

Next, consider a signaling equilibrium in which the sender benefits from making early offers for any outside option value (or a sender-preferred equilibrium) with interval structure $\{V_i\}_{i\in\mathbb{N}}$ and sender offer (v_i^s, t_i^s) if $v^o \in V_i$. The expected welfare of the receiver in this equilibrium is

$$U^* = \sum_{i=1}^{\infty} \int_{\underline{v}_i}^{\bar{v}_i} f(v)(v_i^s + t_i^s)\,\mathrm{d}v, \tag{20}$$

and the expected welfare of the sender equals

$$G^* = \sum_{i=1}^{\infty} \int_{\underline{v}_i}^{\bar{v}_i} f(v)(1 - v_i^s - t_i^s)\,\mathrm{d}v. \tag{21}$$

From U_0^*, G_0^*, U^*, and G^* we obtain the following results. First, if gain-loss sensations do not matter for welfare judgments, $\pi = 0$, then signaling through early offers has no impact on aggregated welfare. Persuading the receiver to accept an inferior offer only redistributes surplus from the receiver to the sender.

Second, this changes as soon as gain-loss sensations are taken into account for welfare judgments, $\pi > 0$. In this case, signaling through early offers increases aggregated welfare. The reason for this is that, in the considered equilibrium, early offers eliminate all gain-loss sensations in period 2. Formally, the increase in welfare is given by the expected gain-loss sensations on the right-hand side of equation (18).

Third, for any given value $\pi > 0$, signaling through early offers even implies a Pareto-improvement if $\eta(\lambda - 1)$ is large enough. Observe from equation (18) that U_0^* becomes negative if $\eta(\lambda - 1)$ is large enough, while U^* is strictly positive. Intuitively, this means that the receiver also benefits from obtaining early offers as they eliminate the uncertainty about future outcomes. If this benefit is large enough, both parties are strictly better off from signaling through early offers.

7 Conclusion

In many bargaining settings, parties gather information over time so that initially asymmetric information about possible options becomes symmetric. We showed in this paper that, in this situation, it can be optimal for the better-informed party to make an early offer to an opponent, in particular, if this opponent has reference-dependent loss-averse preferences. The early offer can credibly reveal information and allow the receiver to attain peace of mind at an early stage by planning its acceptance. This enables the sender to persuade the receiver to accept an offer that is inferior to her outside option, even if she has all payoff-relevant information at the decision stage. There would be no such scope for persuasion through signaling if the receiver had standard preferences.

The analysis highlighted several factors for when the sender can persuade the receiver to accept an inferior offer. The offer needs to have features that outside options do not have, so that giving up these features creates loss sensations through the attachment effect. Next, early offers must be made in a way so that some uncertainty about the value of the outside option remains. Through the uncertainty effect it then can be optimal for the receiver to plan acceptance of an offer that with certainty is inferior to her outside option. Therefore, an equilibrium with persuasion through signaling may exhibit an interval structure, just like the equilibria of cheap-talk games (Crawford and Sobel, 1982).

Our results of course depend on the point in time when the receiver decides on a plan that determines her reference point. We assumed that the receiver chooses the plan after observing the sender's offer, but before learning the value of her outside option. The scope for persuasion through signaling may be different when the receiver already has a plan (and hence a reference point) in her mind when the sender approaches her with his offer. If the receiver already expects to choose the outside option regardless of its value, it may be more difficult (or impossible) to convince her through signaling to accept an offer that with certainty has less consumption utility than the outside option. However, if both the sender's offer and the outside option come as a surprise, there may again be scope for persuasion through signaling, depending on how the reference point is adjusted dynamically. Note that if the receiver expects a zero outcome with certainty in both regular value and transfer dimension (before the sender makes an offer), then – according to the personal equilibrium – she has to choose either the offer or the outside option, provided that both opportunities offer a strictly positive outcome in one dimension and a non-negative outcome in the other dimension.

Our setup can be extended in several directions. The literature on persuasion has examined a variety of settings that could be enriched by taking loss aversion into account, for example, settings with multiple senders, different incentive structures, or incentives to acquire information. The results of the present paper should be helpful for this analysis.

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A Appendix

Proof of Lemma 1. Consider any offer (v^s, t^s) with $v^s + t^s \leq \underline{v}$ and $v^s \leq \underline{v}$ that the sender makes if and only if $v_o \in V \subset [0, 1]$. Let $\hat{F}$ be the updated distribution over outside option values when the receiver observes (v^s, t^s). Consider a cut-off plan σ^r where for some $v^* \in [\underline{v}, \bar{v}]$ the receiver accepts (v^s, t^s) if $v^o \in [\underline{v}, v^*]$ and rejects (v^s, t^s) if $(v^*, \bar{v}]$. After observing (v^s, t^s), the receiver's expected utility from σ^r equals

$$\begin{aligned}\mathbb{E}_{\hat{F}}[U_R(\sigma^r(v^o, v^s, t^s) \mid G^v, G^t)] &= \hat{F}(v^*)(v^s + t^s) + \int_{v^*}^{\bar{v}} \hat{f}(v) v \, \mathrm{d}v \\ &\quad -\eta(\lambda - 1)\hat{F}(v^*)(1 - \hat{F}(v^*)) \, |t^s| \\ &\quad -\eta(\lambda - 1)\hat{F}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} \\ &\quad -\eta(\lambda - 1) \int_{v^*}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \end{aligned} \tag{22}$$

We show that this term is maximal either at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both points. For this, we differentiate the receiver's expected utility with respect to v^*:

$$\begin{aligned}\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} &= \hat{f}(v^*)(v^s + t^s) - \hat{f}(v^*)v^* \\ &\quad -\eta(\lambda - 1)\hat{f}(v^*)(1 - 2\hat{F}(v^*)) \, |t^s| \\ &\quad -\eta(\lambda - 1)\left[\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s) \, \mathrm{d}\tilde{v} - \hat{F}(v^*)\hat{f}(v^*)(v^* - v^s)\right] \\ &\quad +\eta(\lambda - 1)\hat{f}(v^*) \int_{v^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^*) \, \mathrm{d}\tilde{v}. \end{aligned} \tag{23}$$

We can simplify this to

$$\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*} = -\hat{f}(v^*)\left[(v^* - v^s - t^s) + \eta(\lambda - 1)(1 - 2\hat{F}(v^*))(v^* - v^s + |t^s|)\right]. \tag{24}$$

Since $v^s + t^s \leq \underline{v} \leq v^*$, this term is strictly negative for all $v^* > \underline{v}$ with $\hat{F}(v^*) \leq \frac{1}{2}$ and $\hat{f}(v^*) > 0$. Denote by $\Gamma(v^*)$ the term in the squared brackets in (24). The derivative $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ is positive (negative) if and only if $\Gamma(v^*)$ is negative (positive). Consider the derivative

$$\frac{\partial \Gamma(v^*)}{\partial v^*} = 1 + \eta(\lambda - 1)[-2\hat{f}(v^*)(v^* - v^s + |t^s|) + (1 - 2\hat{F}(v^*))]. \tag{25}$$

Since f is weakly increasing on the unit interval, $\hat{f}$ weakly increases in v^* on its support. Hence, the right-hand side of equation (25) strictly decreases in v^*. If $\frac{\partial \Gamma(v^*)}{\partial v^*}$ is negative at $v^* = v^{**}$, it is negative for all $v^* > v^{**}$. By the statement above, if $\frac{\partial \mathbb{E}_{\hat{F}}[\cdot]}{\partial v^*}$ becomes positive at some $v^* = v^{**}$, it remains positive for all $v^* > v^{**}$ with $\hat{f}(v^*) > 0$, which yields us the result. □

Proof of Proposition 1. The proof proceeds in four steps. We prove the first statement of Proposition 1 in Step 1 to Step 3 and the second statement of Proposition 1 in Step 4.

Step 1. Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (0, 1]$ and suppose σ^s is such that the sender makes the offer (v_i^s, t_i^s) to the receiver if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ such that, in period 1, the receiver prefers the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" to any other cut-off plan. Lemma 1 implies that the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" is the payoff-maximizing cut-off plan for the receiver if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected utility from the latter plan equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v. \tag{26}$$

Note that $\hat{f}(v) = \frac{f(v)}{F(\bar{v}_i) - F(\underline{v}_i)}$. If $\hat{f}$ is the uniform distribution on V_i, we have $\hat{f}(v) = \frac{1}{\bar{v}_i - \underline{v}_i}$. Since f is weakly increasing on the unit interval, $\hat{f}$ is weakly increasing on V_i. Consequently, we have $\hat{f}(\underline{v}_i)(\bar{v}_i - \underline{v}_i) \leq 1$ and $\hat{f}(\bar{v}_i)(\bar{v}_i - \underline{v}_i) \geq 1$ and therefore

$$\hat{f}(\underline{v}_i) = \frac{f(\underline{v}_i)}{F(\bar{v}_i) - F(\underline{v}_i)} \leq \frac{1}{\bar{v}_i - \underline{v}_i} \leq \frac{f(\bar{v}_i)}{F(\bar{v}_i) - F(\underline{v}_i)} = \hat{f}(\bar{v}_i). \tag{27}$$

Further, we can estimate

$$f(\underline{v}_i) \geq f(\bar{v}_i) - (\bar{v}_i - \underline{v}_i) \max f'. \tag{28}$$

Define

$$\kappa(\bar{v}_i, \underline{v}_i) = \frac{f(\bar{v}_i)}{f(\bar{v}_i) - (\bar{v}_i - \underline{v}_i) \max f'}. \tag{29}$$

Recall that f' is bounded. Hence, for given values $f(\bar{v}_i)$ and $\max f'$, we have $\kappa(\bar{v}_i, \underline{v}_i) \geq 1$ if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$ (in the following, we therefore assume that $\kappa(\bar{v}_i, \underline{v}_i) \geq 1$) and $\kappa(\bar{v}_i, \underline{v}_i) \to 1$ for $\underline{v}_i \to \bar{v}_i$. Using the inequalities in (27) and (28), we can estimate for $v \in V_i$ that

$$\hat{f}(v) \geq \frac{f(\underline{v}_i)}{F(\bar{v}_i) - F(\underline{v}_i)} \geq \frac{f(\underline{v}_i)}{f(\bar{v}_i)(\bar{v}_i - \underline{v}_i)} \geq \frac{f(\bar{v}_i) - (\bar{v}_i - \underline{v}_i) \max f'}{f(\bar{v}_i)(\bar{v}_i - \underline{v}_i)} = \frac{1}{\kappa(\bar{v}_i, \underline{v}_i)} \frac{1}{\bar{v}_i - \underline{v}_i} \tag{30}$$

and

$$\hat{f}(v) \leq \frac{f(\bar{v}_i)}{F(\bar{v}_i) - F(\underline{v}_i)} \leq \frac{f(\bar{v}_i)}{f(\underline{v}_i)(\bar{v}_i - \underline{v}_i)} \leq \frac{f(\bar{v}_i)}{(f(\bar{v}_i) - (\bar{v}_i - \underline{v}_i) \max f')(\bar{v}_i - \underline{v}_i)} = \kappa(\bar{v}_i, \underline{v}_i) \frac{1}{\bar{v}_i - \underline{v}_i}. \tag{31}$$

For the two terms in (26) we therefore can estimate

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)v\,\mathrm{d}v \leq \kappa(\bar{v}_i,\underline{v}_i)\frac{1}{\bar{v}_i-\underline{v}_i}\int_{\underline{v}_i}^{\bar{v}_i} v\,\mathrm{d}v = \frac{1}{2}\kappa(\bar{v}_i,\underline{v}_i)(\bar{v}_i+\underline{v}_i) \tag{32}$$

and

$$\begin{aligned}\eta(\lambda-1)\int_{\underline{v}_i}^{\bar{v}_i}\hat{f}(v)\int_{v}^{\bar{v}_i}\hat{f}(\tilde{v})(\tilde{v}-v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v &\geq \eta(\lambda-1)\frac{1}{\kappa(\bar{v}_i,\underline{v}_i)^2}\frac{1}{(\bar{v}_i-\underline{v}_i)^2}\int_{\underline{v}_i}^{\bar{v}_i}\int_{v}^{\bar{v}_i}(\tilde{v}-v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v \\ &= \eta(\lambda-1)\frac{1}{6}\frac{1}{\kappa(\bar{v}_i,\underline{v}_i)^2}(\bar{v}_i-\underline{v}_i).\end{aligned} \tag{33}$$

We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we have

$$\underline{v}_i > \frac{1}{2}\kappa(\bar{v}_i,\underline{v}_i)(\bar{v}_i+\underline{v}_i) - \eta(\lambda-1)\frac{1}{6}\frac{1}{\kappa(\bar{v}_i,\underline{v}_i)^2}(\bar{v}_i-\underline{v}_i). \tag{34}$$

We rewrite this inequality as

$$\eta(\lambda-1)\frac{1}{3}(\bar{v}_i-\underline{v}_i) > \kappa(\bar{v}_i,\underline{v}_i)^3(\bar{v}_i+\underline{v}_i) - 2\kappa(\bar{v}_i,\underline{v}_i)^2\underline{v}_i. \tag{35}$$

Note that both the left- and right-hand side of inequality (35) are strictly positive for $\bar{v}_i > \underline{v}_i$ and converge to zero for $\underline{v}_i \to \bar{v}_i$. We show that the term on the left-hand side falls quicker in $\underline{v}_i$ than the term on the right-hand side as long as $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, which implies the desired statement. The derivative of the term on the left-hand side with respect to $\underline{v}_i$ equals $-\eta(\lambda-1)\frac{1}{3}$, while the derivative of the term on the right-hand side with respect to $\underline{v}_i$ is

$$\frac{\partial\kappa(\bar{v}_i,\underline{v}_i)}{\partial\underline{v}_i}\left[3\kappa(\bar{v}_i,\underline{v}_i)^2(\bar{v}_i+\underline{v}_i) - 4\kappa(\bar{v}_i,\underline{v}_i)\underline{v}_i\right] + \kappa(\bar{v}_i,\underline{v}_i)^3 - 2\kappa(\bar{v}_i,\underline{v}_i)^2. \tag{36}$$

Since we have

$$\lim_{\underline{v}_i\to\bar{v}_i}\left[\frac{\partial\kappa(\bar{v}_i,\underline{v}_i)}{\partial\underline{v}_i}\right] = -\max f' \tag{37}$$

the term in (36) converges to $-2\bar{v}_i\max f' - 1$ for $\underline{v}_i \to \bar{v}_i$. By the assumption on the loss aversion parameters, $\eta(\lambda-1) > 3(1+2\max f')$, we have

$$-\eta(\lambda-1)\frac{1}{3} < -2\bar{v}_i\max f' - 1. \tag{38}$$

Hence, if $\underline{v}_i$ is close enough to $\bar{v}_i$, we can find values (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ such that, in period 1, the receiver prefers the plan "accept (v_i^s, t_i^s) if $v^o \in V_i$" to any other cut-off plan.

Step 2. Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (0,1]$ and suppose σ^s is such that the sender makes the offer (v_i^s, t_i^s) to the receiver if and only if $v^o \in V_i$. We show that we can choose (v_i^s, t_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ such that the receiver's PPE specifies to accept this offer whenever $v^o \in V_i$ and to reject it when $v^o > \bar{v}_i$, provided that $\underline{v}_i$ is sufficiently close to $\bar{v}_i$. In period 2, the receiver's utility from following this plan is $v_i^s + t_i^s$, while her utility from choosing the outside option equals $v^o + \eta(v^o - v_i^s) - \eta\lambda t_i^s$. As we discuss in the main text (in the paragraph on the attachment effect after Proposition 1), if $\underline{v}_i > \frac{1+\eta}{1+\eta\lambda}\bar{v}_i$, we can choose for any total value $w_i^s < \underline{v}_i$ that is sufficiently close to $\underline{v}_i$ an offer (v_i^s, t_i^s) with $v_i^s + t_i^s = w_i^s$ and $t_i^s \geq 0$ such that

$$v_i^s + t_i^s = \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s. \tag{39}$$

The equality implies that the receiver is indifferent between accepting and rejecting the sender's offer (v_i^s, t_i^s) in period 2 when $v^o = \bar{v}_i$. By Step 1, if additionally $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then, in a PPE, the receiver accepts offer (v_i^s, t_i^s) if $v^o \in V_i$ and rejects it if $v^o > \bar{v}_i$. This completes the proof of the statement.

Step 3. We construct a signaling equilibrium with the desired property. Consider the interval $(v_L, 1]$ for any given $0 < v_L < 1$. By Step 1, if the value $\varepsilon > 0$ is sufficiently small, then we can choose for any $\bar{v}_i \in (v_L, 1]$ a value $\underline{v}_i^*$ with $\frac{1+\eta}{1+\eta\lambda}\bar{v}_i < \underline{v}_i^* < \bar{v}_i - \varepsilon$ such that for any $\underline{v}_i \in [\underline{v}_i^*, \bar{v}_i)$ we have

$$\underline{v}_i > \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)v \, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v) \, \mathrm{d}\tilde{v} \, \mathrm{d}v, \tag{40}$$

i.e., the value $\underline{v}_i$ strictly exceeds the payoff from the plan "accept the outside option if $v^o \in V_i = (\underline{v}_i, \bar{v}_i]$." Pick a function $\underline{v}_i^*(\bar{v}_i)$, defined on the interval $(v_L, 1]$, with the following property: For each value $\bar{v}_i \in (v_L, 1]$, we have $\underline{v}_i^*(\bar{v}_i) < \bar{v}_i - \varepsilon$ for some sufficiently small $\varepsilon > 0$ and $\underline{v}_i^*(\bar{v}_i)$ strictly exceeds the right-hand side of inequality (40) when $\underline{v}_i = \underline{v}_i^*(\bar{v}_i)$. Further, define

$$R = \sup\left[\frac{\underline{v}_i^*(\bar{v}_i)}{\bar{v}_i}; \bar{v}_i \in (v_L, 1]\right]. \tag{41}$$

Note that, by construction, we have $R < 1$. We can now choose a finite sequence of half-open intervals $V_1, V_2, ..., V_n$ with $\underline{v}_i = R\bar{v}_i$ and $\bar{v}_{i+1} = \underline{v}_i$ for each $i \in \{1, ..., n-1\}$, and the property that $(v_L, 1] \subset \bigcup_{i=1}^n V_i \subset (0,1]$, as well as a sequence of offers $\{(v_i^s, t_i^s)\}_{i=1}^n$ so that $v_i^s + t_i^s$ strictly decreases in i and we have $0 < v_i^s + t_i^s < \underline{v}_i$ for all $i \in \{1, ..., n\}$. Suppose σ^s is such that the sender offers (v_i^s, t_i^s) if and only if $v^o \in V_i$. By Step 2, the sequence of offers can be chosen so that, at σ^s, it is a PPE for the receiver to accept the sender's offer (v_i^s, t_i^s) if $v^o \leq \bar{v}_i$ and to reject it otherwise.

Recall that we chose an arbitrary lower bound $v_L \in (0, 1)$ for the interval $(v_L, 1]$. Then we constructed a finite sequence of intervals $\{V_i\}_{i=1}^n$ so that $(v_L, 1] \subset \bigcup_{i=1}^n V_i$. Therefore, we can apply the arguments above repeatedly to construct an infinite sequence of intervals $\{V_i\}_{i\in\mathbb{N}}$ with $\bigcup_{i=1}^{\infty} V_i = (0, 1]$ and offers $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ with the above mentioned properties. Finally, for any offer (v^s, t^s) that is not an element of the set $\{(v_i^s, t_i^s)\}_{i\in\mathbb{N}}$ we specify that in period 1 the receiver believes that her outside option value is $v^o = 1$ with certainty. It is then optimal for her to reject an offer (v^s, t^s) in period 2 if $v^s + t^s \le v^o$. Given this behavior, it is indeed optimal for the sender to offer (v_i^s, t_i^s) if and only if $v^o \in V_i$. This completes the proof of the first statement of Proposition 1.

Step 4. We prove the second statement of Proposition 1. Note that, on any equilibrium path, it must be the case that the receiver accepts the sender's offer if $v^o < 1$. Hence, for any two values $v, \hat{v} \in [0, 1)$ with $v > \hat{v}$ the following must hold: Suppose the sender offers the total value (the sum of regular value and transfer) w if $v^o = v$ and $\hat{w}$ if $v^o = \hat{v}$. Then we must have $w \ge \hat{w}$. Otherwise, the sender could deviate profitably at $v^o = \hat{v}$ by making the same offer as for $v^o = v$ since the receiver would accept it. Given this result, we can make the following observation: Assume by contradiction that there exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the sender makes offers with varying total value, $w \ne \hat{w}$. The receiver would then be able to infer her outside option value from these offers so that the sender cannot persuade her to accept an inferior offer, a contradiction. Hence, an equilibrium in which the sender benefits from making early offers at all outside option values $v^o > 0$ must be characterized by a sequence of disjoint intervals $\{V_i\}_{i\in\mathbb{N}}$ and values $\{w_i\}_{i\in\mathbb{N}}$, so that the sender makes an offer (v^s, t^s) with total value $v^s + t^s = w_i < \underline{v}_i$ and non-zero transfer t^s if $v^o \in V_i$; the receiver accepts this offer. □

Proof of Proposition 2. On any equilibrium path, the receiver accepts the sender's offer if $v^o < 1$. Otherwise, the sender could deviate profitably by offering $(v^s, t^s) = (v^o, 0)$, which the receiver would accept. As in Step 4 of the proof of Proposition 1, we can show that, if the sender offers the total value w if $v^o = v < 1$ and the total value $\hat{w}$ if $v^o = \hat{v} < v$, then we must have $w \ge \hat{w}$. Assume by contradiction that, in a sender-preferred equilibrium σ, there exists an interval $V = (v_L, v_H) \subset [0, 1]$ so that for any two outside option values $v, \hat{v} \in V$ the sender makes offers with varying total value, $w \ne \hat{w}$. The receiver would then be able to infer her outside option value from these offers so that the total value of a sender offer must equal the outside option value for each $v^o \in V$. We then can find an alternative equilibrium σ' that is identical to σ except that there is an interval of outside option values $(v'_L, v'_H) \subset (v_L, v_H)$ at which the sender makes an offer with total value v'_L. This can be shown by using similar arguments as in Step 1 and Step 2 of the proof of Proposition 1. The sender's expected payoff in equilibrium σ' then strictly exceeds that in equilibrium σ, a contradiction. Finally, note that the receiver accepts a sender offer (v^s, t^s) with total value $v^s + t^s$ strictly below her outside option value v^o in period 2 only if $t^s \neq 0$. This completes the proof. □

Proof of Lemma 2. Suppose the sender makes, for some interval $V \subset [0, 1]$, the offer (v^s, t^s, ξ^s) with $v^s + t^s \leq \underline{v}$ and $v^s \leq \underline{v}$ if and only if $v_o \in V$. Let $\hat{F}$ be the updated distribution over outside option values when the receiver observes (v^s, t^s, ξ^s). Since f is weakly increasing on the unit interval, $\hat{f}$ weakly increases on its support. Consider a cut-off plan σ^r characterized by two values $v_1^*, v_2^* \in [\underline{v}, \bar{v}]$ with $v_1^* \leq v_2^*$ that has the following features: In state 1, the receiver accepts (v^s, t^s, ξ^s) if $v^o \in [\underline{v}, v_1^*]$ and rejects (v^s, t^s, ξ^s) if $(v_1^*, \bar{v}]$. In state 2, the receiver accepts (v^s, t^s, ξ^s) if $v^o \in [\underline{v}, v_2^*]$ and rejects (v^s, t^s, ξ^s) if $(v_2^*, \bar{v}]$. For the case $v_1^* \geq v_2^*$ we can use the same arguments as below (we mention this case at a later stage).

Depending on the cut-off values v_1^* and v_2^* the receiver expects to experience gain-loss sensations in the x- and y-dimension. The size of these gain-loss sensations depends on the value of ξ^s relative to ξ^o. To write down the expected gain-loss sensations in the extra-dimensions, we introduce the following definitions: ξ^s belongs to Category (i) if $\xi^s \geq \xi^o + \xi$, to Category (ii) if $\xi^o + \xi > \xi^s \geq \xi^o - \xi$, and to Category (iii) if $\xi^o - \xi > \xi^s$. Further, we define the values L_1 and L_2 (gain-loss sensations in the x- and y-dimension) as indicated in the following table:

	Category (i)	Category (ii)	Category (iii)
L_1	$\xi^s - \xi^o - \xi$	$\xi^o - \xi^s + \xi$	$\xi^o - \xi^s + \xi$
L_2	$\xi^s - \xi^o + \xi$	$\xi^s - \xi^o + \xi$	$\xi^o - \xi^s - \xi$

Note that we have $L_1 + L_2 - 2\xi \geq 0$ for all categories. We will use this fact at several instances.

After observing (v^s, t^s, ξ^s), the receiver's expected utility from strategy σ^r equals

$$\begin{aligned}
\mathbb{E}_{\hat{F}}[U_R(.)] = & \left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(v^s + t^s) + \frac{1}{2}\int_{v_1^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v + \frac{1}{2}\int_{v_2^*}^{\bar{v}} \hat{f}(v)v \,\mathrm{d}v \\
& -\eta(\lambda-1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(1 - \frac{1}{2}\hat{F}(v_1^*) - \frac{1}{2}\hat{F}(v_2^*)\right)|t^s| \\
& -\eta(\lambda-1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v}\right) \\
& -\eta(\lambda-1)\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(v)\left(\frac{1}{2}\int_{v}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\right)\mathrm{d}v \\
& -\eta(\lambda-1)\int_{v_2^*}^{\bar{v}} \hat{f}(v)\int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\,\mathrm{d}\tilde{v}\,\mathrm{d}v \\
& -\eta(\lambda-1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(1 - \hat{F}(v_1^*))L_1 \\
& -\eta(\lambda-1)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(1 - \hat{F}(v_2^*))L_2 \\
& -\eta(\lambda-1)(1 - \hat{F}(v_1^*))(1 - \hat{F}(v_2^*))\xi. \qquad (42)
\end{aligned}$$

The first line is the receiver's consumption utility, the second line is the receiver's expected gain-loss utility in the transfer dimension, line three to five capture the receiver's expected gain-loss utility in the regular value dimension, and line six to eight capture the receiver's expected gain-loss utility in the two extra-dimensions. The rest of the proof proceeds in four steps. Steps 1 to 3 taken together imply statement (a). In Step 4, we prove statement (b).

Step 1. We show that for given $v_2^* > \underline{v}$ the value in (42) is maximal at $v_1^* = \underline{v}$ or at $v_1^* = v_2^*$ or at both values. The first derivative of (42) with respect to v_1^* equals

$$\begin{aligned}
\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} = & -\frac{1}{2}\hat{f}(v_1^*)(v_1^* - v^s - t^s) \\
& -\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)(1 - \hat{F}(v_1^*) - \hat{F}(v_2^*))\,|t^s| \\
& +\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\hat{F}(v_1^*) + \frac{1}{2}\hat{F}(v_2^*)\right)(v_1^* - v^s) \\
& -\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\,\mathrm{d}\tilde{v}\right) \\
& +\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\left(\frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(\tilde{v} - v_1^*)\,\mathrm{d}\tilde{v} + \int_{v_2^*}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v_1^*)\,\mathrm{d}\tilde{v}\right) \\
& -\eta(\lambda-1)\frac{1}{2}\hat{f}(v_1^*)\Big((1 - 2\hat{F}(v_1^*) - \hat{F}(v_2^*))L_1 + (1 - \hat{F}(v_2^*))(L_2 - 2\xi)\Big), \qquad (43)
\end{aligned}$$

which can be simplified to

$$\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_1^*} = -\frac{1}{2}\hat{f}(v_1^*)\Big[(v_1^* - v^s - t^s) + \eta(\lambda - 1) \times \Big((1 - \hat{F}(v_1^*) - \hat{F}(v_2^*))\,|t^s| + (1 - \hat{F}(v_1^*) - \hat{F}(v_2^*))(v_1^* - v^s) + \Big((1 - 2\hat{F}(v_1^*) - \hat{F}(v_2^*))L_1 + (1 - \hat{F}(v_2^*))(L_2 - 2\xi)\Big)\Big)\Big]. \quad (44)$$

Denote the term in squared brackets by $\Gamma_1(v_1^*, v_2^*)$. The second derivative of (42) with respect to v_1^* equals

$$\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial (v_1^*)^2} = -\frac{1}{2}\hat{f}'(v_1^*)\Gamma_1(v_1^*, v_2^*) - \frac{1}{2}\hat{f}(v_1^*)\Big[1 + \eta(\lambda - 1) \times \Big(-\hat{f}(v_1^*)(v_1^* - v^s + |t^s| + 2L_1) + (1 - \hat{F}(v_1^*) - \hat{F}(v_2^*))\Big)\Big]. \quad (45)$$

Fix v_2^* and assume by contradiction that $\mathbb{E}_{\hat{F}}[U_R(.)]$ has a local maximum at $\hat{v}_1^* \in (\underline{v}, v_2^*)$. Note that the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to v_1^* is strictly negative at $v_1^* = \underline{v}$ (to show this, we use the fact that $L_1 + L_2 - 2\xi \geq 0$ for all categories). Hence, there must be a local minimum of $\mathbb{E}_{\hat{F}}[U_R(.)]$ at some value $\tilde{v}_1^* \in (\underline{v}, \hat{v}_1^*)$. At a local maximum or minimum, we must have $\Gamma_1(., v_2^*) = 0$. Thus, the term in squared brackets on the right-hand side of equation (45) must be negative at $v_1^* = \tilde{v}_1^*$ and positive at $v_1^* = \hat{v}_1^*$. This implies that

$$\hat{f}(\tilde{v}_1^*)(\tilde{v}_1^* - v^s + |t^s| + 2L_1) + \hat{F}(\tilde{v}_1^*) > \hat{f}(\hat{v}_1^*)(\hat{v}_1^* - v^s + |t^s| + 2L_1) + \hat{F}(\hat{v}_1^*), \quad (46)$$

which contradicts the fact that $\hat{f}$ weakly increases on its support and $\hat{v}_1^* > \tilde{v}_1^*$. This completes the proof of the statement.

Step 2. We show that at $v_1^* = \underline{v}$ the expected payoff in (42) is maximal at $v_2^* = \underline{v}$ or at $v_2^* = \bar{v}$ or at both values. The first derivative of (42) with respect to v_2^* equals

$$\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v_2^*} = -\frac{1}{2}\hat{f}(v_2^*)\Big[(v_2^* - v^s - t^s) + \eta(\lambda - 1)\Big((1 - \hat{F}(v_1^*) - \hat{F}(v_2^*))\,|t^s| + \Big(1 - \frac{1}{2}\hat{F}(v_1^*) - \frac{3}{2}\hat{F}(v_2^*)\Big)(v_2^* - v^s) + \frac{1}{2}\int_{v_1^*}^{v_2^*} \hat{f}(\tilde{v})(2\tilde{v} - v^s - v_2^*)\,\mathrm{d}\tilde{v} + \Big((1 - 2\hat{F}(v_2^*) - \hat{F}(v_1^*))L_2 + (1 - \hat{F}(v_1^*))(L_1 - 2\xi)\Big)\Big)\Big]. \quad (47)$$

Denote the term in squared brackets by $\Gamma_2(v_1^*, v_2^*)$. The second derivative of (42) with respect to v_2^* equals

$$\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial(v_2^*)^2} = -\frac{1}{2}\hat{f}'(v_2^*)\Gamma_2(v_1^*, v_2^*) - \frac{1}{2}\hat{f}(v_2^*)\Big[1 + \eta(\lambda - 1) \times \Big(-\hat{f}(v_2^*)\left(v_2^* - v^s + |t^s| + 2L_2\right) + \Big(1 - 2\hat{F}(v_2^*)\Big)\Big)\Big]. \quad (48)$$

Fix $v_1^* = \underline{v}$ and assume by contradiction that $\mathbb{E}_{\hat{F}}[U_R(.)]$ has a local maximum at $\hat{v}_2^* \in (\underline{v}, \bar{v})$. Note that, at $v_1^* = \underline{v}$ and $v_2^* = \underline{v}$, the first derivative of $\mathbb{E}_{\hat{F}}[U_R(.)]$ with respect to v_2^* is strictly negative (to show this, we again use the fact that $L_1 + L_2 - 2\xi \geq 0$ for all categories). Hence, there must be a local minimum of $\mathbb{E}_{\hat{F}}[U_R(.)]$ at some value $\tilde{v}_2^* \in (\underline{v}, \hat{v}_2^*)$. At a local maximum or minimum, we must have $\Gamma_2(v_1^*, .) = 0$. Therefore, the term in squared brackets on the right-hand side of equation (48) must be negative at $v_2^* = \tilde{v}_2^*$ and positive at $v_2^* = \hat{v}_2^*$. As in Step 1, we can show that this contradicts the fact that $\hat{f}$ weakly increases on its support and $\hat{v}_2^* > \tilde{v}_2^*$. This completes the proof of the statement.

Step 3. We consider the set of cut-off plans with $v_1^* = v_2^* = v^*$ and show that the expected payoff in (42) for these plans is maximal at $v^* = \underline{v}$ or at $v^* = \bar{v}$ or at both values. The first derivative of the expected payoff in (42) with respect to v^* is

$$\frac{\partial \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial v^*} = -\hat{f}(v^*)\Big[(v^* - v^s - t^s) + \eta(\lambda - 1) \times \Big((1 - 2\hat{F}(v^*))(v^* - v^s + |t^s| + L_1 + L_2) - (1 - \hat{F}(v^*))2\xi\Big)\Big]. \quad (49)$$

Let $\Gamma_3(v^*)$ be the term in squared brackets on the right-hand side of equation (49). The second derivative of (42) with respect to v^* equals

$$\frac{\partial^2 \mathbb{E}_{\hat{F}}[U_R(.)]}{\partial(v^*)^2} = -\hat{f}'(v^*)\Gamma_3(v^*) - \hat{f}(v^*)\Big[1 + \eta(\lambda - 1) \times \Big(-2\hat{f}(v^*)(v^* - v^s + |t^s| + L_1 + L_2 - \xi) + (1 - 2\hat{F}(v^*))\Big)\Big]. \quad (50)$$

We can now apply the same arguments as in Step 1 and Step 2 to prove that $\mathbb{E}_{\hat{F}}[U_R(.)]$ has no local maximum at some value $\hat{v}^* \in (\underline{v}, \bar{v})$. This completes the proof of the statement.

Step 4. Steps 1 to 3 taken together and repeating them for the case with cut-off values $v_1^* \geq v_2^*$ prove statement (a). In the following, we prove statement (b). If $\xi^s = \xi^o$, we are in the domain of Category (ii) and we have $L_1 = L_2 = \xi$. From Steps 1 to 3 it follows that only the following three cut-off plans potentially maximize the expected payoff in equation (42): a plan with $v_1^* = v_2^* = \bar{v}$, a plan with $v_1^* = v_2^* = \underline{v}$, and a plan with $v_1^* = \underline{v}$ and $v_2^* = \bar{v}$ (or with $v_1^* = \bar{v}$ and $v_2^* = \underline{v}$, which in terms of expected payoff is equivalent). We show that the expected payoff from the last plan is always strictly smaller than the expected payoff of the first or second plan.

The expected payoff from the first plan is $U_1 = v^s + t^s$. The expected payoff from the second plan is

$$U_2 = \int_{\underline{v}}^{\bar{v}} \hat{f}(v)v\, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v - \eta(\lambda - 1)\xi, \tag{51}$$

and the expected payoff from the third plan is

$$\begin{aligned} U_3 \;\; = \;\; & \frac{1}{2}(v^s + t^s) + \frac{1}{2} \int_{\underline{v}}^{\bar{v}} \hat{f}(v)v\, \mathrm{d}v - \eta(\lambda - 1)\frac{1}{4}|t^s| - \eta(\lambda - 1)\frac{1}{4} \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\, \mathrm{d}\tilde{v} \\ & -\eta(\lambda - 1)\frac{1}{4} \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v - \eta(\lambda - 1)\frac{1}{2}\xi. \end{aligned} \tag{52}$$

Note that

$$\int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{v}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v < \int_{\underline{v}}^{\bar{v}} \hat{f}(v) \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\, \mathrm{d}\tilde{v}\, \mathrm{d}v = \int_{\underline{v}}^{\bar{v}} \hat{f}(\tilde{v})(\tilde{v} - v^s)\, \mathrm{d}\tilde{v}. \tag{53}$$

We can use this to show that if $U_1 \geq U_2$, then we also have $U_1 > U_3$; and if $U_2 \geq U_1$, then we also have $U_2 > U_3$, which completes the proof. □

Proof of Proposition 3. The proof proceeds in two steps.

Step 1. Assume that $\frac{\eta(\lambda-1)}{1+\eta}\xi < 1$. For the case $\frac{\eta(\lambda-1)}{1+\eta}\xi \geq 1$ the proof proceeds in Step 2. Consider an interval $V_i = (\underline{v}_i, \bar{v}_i] \subset (\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ and assume that σ^s is such that the sender makes the offer (v_i^s, t_i^s, ξ_i^s) with $\xi_i^s = \xi^o$ to the receiver if and only if $v^o \in V_i$. We show that if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, then we can choose (v_i^s, t_i^s, ξ_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ such that the receiver's PPE at σ^s specifies to accept this offer whenever $v^o \in V_i$. Lemma 2 statement (b) implies that the plan "accept (v_i^s, t_i^s, ξ_i^s) if $v^o \in V_i$" is the payoff-maximizing cut-off plan for the receiver if its expected payoff exceeds that from the plan "accept the outside option if $v^o \in V_i$." Her expected payoff from the latter plan after observing the offer (v_i^s, t_i^s, ξ_i^s) equals

$$\int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v)v\, \mathrm{d}v - \eta(\lambda - 1) \int_{\underline{v}_i}^{\bar{v}_i} \hat{f}(v) \int_{v}^{\bar{v}_i} \hat{f}(\tilde{v})(\tilde{v} - v)\, \mathrm{d}\tilde{v}\, \mathrm{d}v - \eta(\lambda - 1)\xi. \tag{54}$$

As in Step 1 of the proof of Proposition 1, we can show that this value is (weakly) smaller than

$$\frac{1}{2}\kappa(\bar{v}_i, \underline{v}_i)(\bar{v}_i + \underline{v}_i) - \eta(\lambda - 1)\frac{1}{6}\frac{1}{\kappa(\bar{v}_i, \underline{v}_i)^2}(\bar{v}_i - \underline{v}_i) - \eta(\lambda - 1)\xi \tag{55}$$

if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$. The term $\kappa(\bar{v}_i, \underline{v}_i)$ is defined in equation (29) and has the attributes that $\kappa(\bar{v}_i, \underline{v}_i) \geq 1$ if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$ and $\kappa(\bar{v}_i, \underline{v}_i) \to 1$ for $\underline{v}_i \to \bar{v}_i$. By assumption, we have $\eta(\lambda - 1)\xi > 0$. Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$ and the total value $v_i^s + t_i^s = w_i^s < \underline{v}_i$

is sufficiently close to $\underline{v}_i$, then the payoff-maximizing cut-off plan in period 1 is to accept (v_i^s, t_i^s, ξ_i^s) if $v^o \in V_i$.

We examine next when this plan is consistent with a PPE at σ^s. In period 2, the receiver's payoff from accepting the sender's offer is $v_i^s + t_i^s$, while the payoff from accepting the outside option value is, in both states (due to $\xi_i^s = \xi^o$), equal to

$$v^o + \eta(v^o - v_i^s) - \eta\lambda t_i^s - \eta(\lambda - 1)\xi. \tag{56}$$

The receiver is indifferent between the sender's offer and the outside option at $v^o = \bar{v}_i$ if

$$v_i^s + t_i^s = \bar{v}_i + \eta(\bar{v}_i - v_i^s) - \eta\lambda t_i^s - \eta(\lambda - 1)\xi. \tag{57}$$

We can find values v_i^s, t_i^s with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ that satisfy this equality if

$$\bar{v}_i + \eta\bar{v}_i - \eta\lambda\underline{v}_i - \eta(\lambda - 1)\xi < \underline{v}_i, \tag{58}$$

which is equivalent to the inequality

$$\frac{1+\eta}{1+\eta\lambda}\bar{v}_i - \frac{\eta(\lambda-1)}{1+\eta\lambda}\xi < \underline{v}_i. \tag{59}$$

Hence, if $\underline{v}_i$ is sufficiently close to $\bar{v}_i$, we can find values v_i^s, t_i^s with $v_i^s + t_i^s < \underline{v}_i$ and $t_i^s \geq 0$ such that, in the PPE, the receiver always accepts (v_i^s, t_i^s, ξ_i^s) if $v^o \in V_i$.

Step 2. We can now construct the desired equilibrium. Suppose the sender adopts the following strategy: If $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$, the sender offers (v^s, t^s, ξ^s) to the receiver, with $v^s = 0$, $\xi^s = \xi^o$, and

$$t^s = v^o - \frac{\eta(\lambda-1)}{1+\eta}\xi. \tag{60}$$

Note that $t^s < 0$ if $v^o < \frac{\eta(\lambda-1)}{1+\eta}\xi$. If $v^o > \frac{\eta(\lambda-1)}{1+\eta}\xi$, the sender makes the following offer: The interval $(\frac{\eta(\lambda-1)}{1+\eta}\xi, 1]$ is partitioned by a finite sequence of disjoint half-open intervals $\{V_i\}_{i=1}^n$ so that the sender offers (v_i^s, t_i^s, ξ_i^s) with $v_i^s + t_i^s < \underline{v}_i$ and $\xi_i^s = \xi^o$ to the receiver if $v^o \in V_i$. For each $i = 1, ..., n$, the interval V_i as well as the values v_i^s, t_i^s are chosen such that the receiver is indifferent between the sender's offer and the outside option at $v^o = \bar{v}_i$ in period 2 and accepting on-equilibrium path offers characterizes the receiver's PPE. The result in Step 1 implies that this is possible.

It remains to show that the receiver also accepts the equilibrium offers in a PPE at σ^s if $v^o \leq \frac{\eta(\lambda-1)}{1+\eta}\xi$. Note that the receiver learns her outside option value from these offers. In period

1, planning the acceptance of these offers is optimal for the receiver if

$$t^s \geq v^o - \eta(\lambda - 1)\xi. \tag{61}$$

In period 2, the receiver is indifferent between accepting and rejecting these offers (provided that acceptance has been planned in period 1) if

$$t^s = v^o + \eta v^o + \eta(-t^s) - \eta(\lambda - 1)\xi, \tag{62}$$

which is equivalent to equation (60). The assumption $\eta > 0$ implies that the condition in (61) is satisfied with strict inequality. Finally, assuming optimistic beliefs for out-of-equilibrium offers ensures that no party can deviate profitably. □

Proof of Proposition 4. The proof proceeds in two steps.

Step 1. Assume by contradiction that there is a sender-preferred equilibrium $\sigma = (\sigma^s, \sigma^r, \hat{F})$ that is a separating equilibrium. Suppose further that the sender's strategy σ^s is such that there is an interval $(v_L, v_H) \subset [0, 1]$ with $v_L > \frac{1}{2}$ so that for almost every value $v^o \in (v_L, v_H)$ the sender makes an offer (v^s, t^s, ξ^s) with $v^s + t^s < v^o - \eta(\lambda - 1)\xi$. We assume w.l.o.g. that ξ is small enough such that $v_L - \eta(\lambda - 1)\xi > 0$. We show that, if for given η, λ the parameter ξ is sufficiently small, then we can modify σ so that we obtain an equilibrium which dominates σ in terms of expected payoff for the sender.

Pick any value $v \in (v_L, v_H)$ so that the sender's offer at $v^o = v$ has the above mentioned property. We first show that, in the PPE σ^r, the receiver does not always accept the sender's offer (v^s, t^s, ξ^s) if $v^o = v$. To this end, we show that, at the sender strategy σ^s, there is a PE $\tilde{\sigma}^r$ in which the receiver accepts the outside option with certainty if $v^o = v$ and the sender's offer is (v^s, t^s, ξ^s). Recall the definitions of Category (i) to Category (iii) from the proof of Lemma 2. Suppose in period 1 the receiver plans to accept the outside option after observing offer (v^s, t^s, ξ^s). Her expected payoff then equals $v - \eta(\lambda - 1)\xi$ in period 1 and her realized payoff in period 2 from following this plan also equals $v - \eta(\lambda - 1)\xi$ (in both states). In period 2, her payoff from unexpectedly accepting the sender's offer is (in both states) weakly less than $v^s + t^s - \eta(\lambda - 1)(\xi^s - \xi^o) < v - \eta(\lambda - 1)\xi$ in Category (i); this payoff is weakly less than $v^s + t^s - \eta(\lambda - 1)\xi < v - \eta(\lambda - 1)\xi$ in Category (ii); and it is weakly less than $v^s + t^s - \eta(\lambda - 1)(\xi^o - \xi^s) < v - \eta(\lambda - 1)\xi$ in Category (iii). Hence, at the sender strategy σ^s, there is a PE $\tilde{\sigma}^r$ in which the receiver accepts the outside option with certainty if $v^o = v$ and the sender's offer is (v^s, t^s, ξ^s). Since $v^s + t^s < v - \eta(\lambda - 1)\xi$ and σ^r is a PPE at given σ^s, it therefore cannot be the case that, according to σ^r, the receiver accepts (v^s, t^s, ξ^s) with certainty if $v^o = v$. Further, we must have

$$\frac{1}{2}v + \frac{1}{2}(v^s + t^s) \geq v - \eta(\lambda - 1)\xi, \tag{63}$$

otherwise the receiver would never accept (v^s, t^s, ξ^s) if $v^o = v$ and σ cannot be an equilibrium (note that the receiver's payoff in period 2 from accepting the outside option cannot exceed v).

We now modify the equilibrium σ to make it more profitable for the sender: For any given outside option value $v \in [v_L, v_H]$, the sender makes the offer $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ with $\hat{v}^s = 0$,

$$\hat{t}^s = \frac{1 + \eta}{1 + \eta\lambda}v - \frac{\eta(\lambda - 1)}{1 + \eta\lambda}\xi, \tag{64}$$

and $\hat{\xi}^s = \xi^o$. Note that $\hat{t}^s > 0$ (since we have $v_L - \eta(\lambda - 1)\xi > 0$ by assumption). The offer is chosen such that in period 2 the receiver is indifferent between accepting and rejecting it if in period 1 she planned to accept it with certainty and her outside option value in period 2 is indeed $v^o = v$. There is then a PPE at the modified sender strategy where she accepts this offer if $v^o = v$ and rejects it if $v^o > v$. The sender's payoff from this offer is $1 - \hat{t}^s$. His expected payoff from the original offer (v^s, t^s, ξ^s) is $\frac{1}{2}(1 - (v^s + t^s))$. Due to the inequality in (63), his payoff from the original offer is at most $\frac{1}{2} - \frac{1}{2}v + \eta(\lambda - 1)\xi$. The payoff $1 - \hat{t}^s$ is strictly larger than that if

$$1 - \frac{1 + \eta}{1 + \eta\lambda}v + \frac{\eta(\lambda - 1)}{1 + \eta\lambda}\xi > \frac{1}{2} - \frac{1}{2}v + \eta(\lambda - 1)\xi. \tag{65}$$

Since $v \in [0, 1]$ this inequality is implied by

$$\min\left\{\frac{1}{2}, 1 - \frac{1 + \eta}{1 + \eta\lambda}\right\} > \eta(\lambda - 1)\xi\frac{\eta\lambda}{1 + \eta\lambda}. \tag{66}$$

Note that the left-hand side of this inequality is strictly positive since $\lambda > 1$. Hence, if for given η, λ the parameter ξ is small enough, then at $v^o = v$ the sender's payoff from the modified offer $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ is strictly larger than that from the original offer (v^s, t^s, ξ^s).

We complete the modification of σ: For any given outside option value $v \in (v_H, 1]$, the sender makes the same offer as under σ. For any given outside option value $v \in [0, v_L)$, the sender makes the same offer as under σ, except when the sender's expected payoff from doing so in σ is weakly smaller than his payoff from making the modified offer at $v^o = v_L$; in this case, the sender offers $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ as if $v = v_L$. The receiver's PPE σ^r is adjusted accordingly: At all modified offers the receiver accepts the sender's offer in both states and at all other offers the receiver acts as in the original PPE. By construction, the sender's expected equilibrium payoff in the modified equilibrium is larger than in the original equilibrium. Finally, using optimistic beliefs for off-equilibrium offers renders making such offers unprofitable.

Step 2. Assume by contradiction that there is a sender-preferred equilibrium $\sigma = (\sigma^s, \sigma^r, \hat{F})$ that is a separating equilibrium. We show that, if for given η, λ the parameter ξ is small enough, then we can modify σ so that we obtain an equilibrium with bunching at the highest outside option values which dominates σ in terms of expected payoff for the sender.

By Step 1, if for given η, λ the parameter ξ is small enough, then we can find a value $v_L \in (0, 1)$ so that the expected total value that the sender offers to the receiver in equilibrium σ given that $v^o > v_L$ is at least

$$\int_{v_L}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\xi, \tag{67}$$

where $\hat{f}$ is the density conditional on $v^o \in (v_L, 1]$. We then can find a value w^s so that the following inequalities hold:

$$w^s < \int_{v_L}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\xi, \tag{68}$$

$$w^s \geq \int_{v_L}^{1} v\hat{f}(v)dv - \eta(\lambda - 1)\int_{v_L}^{1} \hat{f}(v)\int_{v}^{1} \hat{f}(\tilde{v})(\tilde{v} - v)d\tilde{v}dv - \eta(\lambda - 1)\xi. \tag{69}$$

We now modify σ as follows: If $v^o \in [v_L, 1]$, the sender makes the offer $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ with total value $\hat{v}^s + \hat{t}^s = w^s$ and $\hat{\xi}^s$ large (or small) enough such that there is no PPE where the receiver accepts $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ in one state and accepts the outside option in another state. For any given outside option value $v^o \in [0, v_L)$, the sender makes the same offer as under σ, except when the sender's expected payoff from doing so in equilibrium σ is weakly smaller than his payoff from making the modified offer at $v^o \in [v_L, 1]$; in this case, the sender offers $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ as if $v^o \in [v_L, 1]$. The condition in (68) ensures that the sender's expected payoff under the modified equilibrium is strictly larger than under the original equilibrium. By Lemma 2 statement (a) and the choice of $\hat{\xi}^s$, the condition in (69) ensures that it is indeed optimal for the receiver in period 1 to plan the acceptance of the sender's offer $(\hat{v}^s, \hat{t}^s, \hat{\xi}^s)$ in period 2 in both states (as long as it does not turn out to be an off-equilibrium offer). Again, using optimistic beliefs for off-equilibrium offers renders making such offers unprofitable. This completes the proof of the statement. □